Autoseg 2013
Opening remarks

Greg Sharp
MGH, Mar 8 2013

Dice's coefficient

- Invented in 1940's for botany applications

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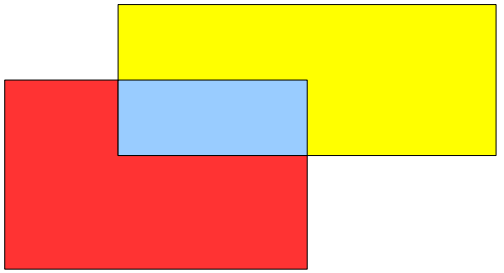

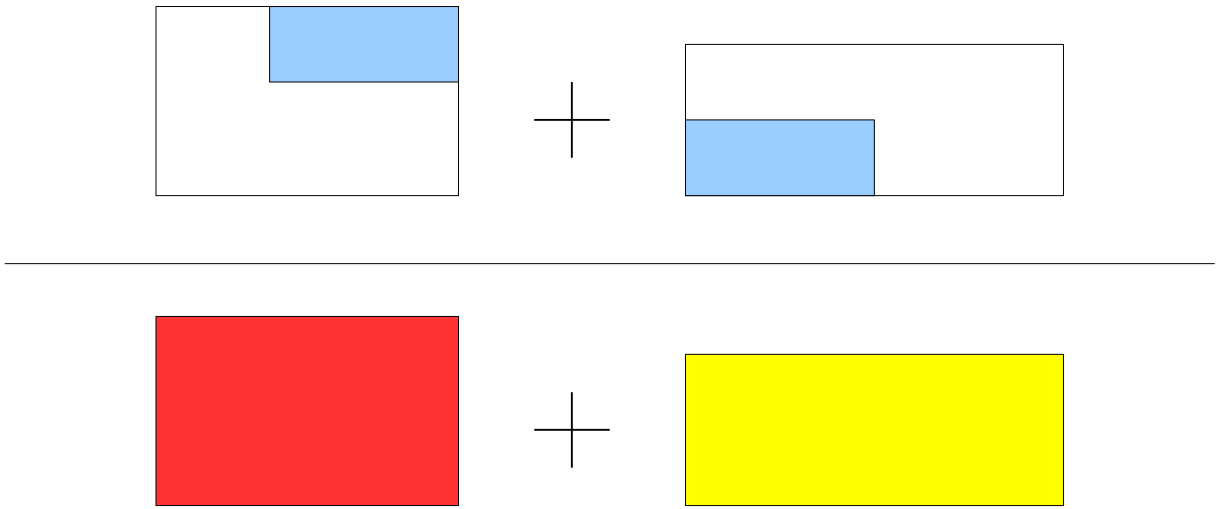

Dice's coefficient

- What is the Dice coefficient when matching a parotid gland with a sphere?

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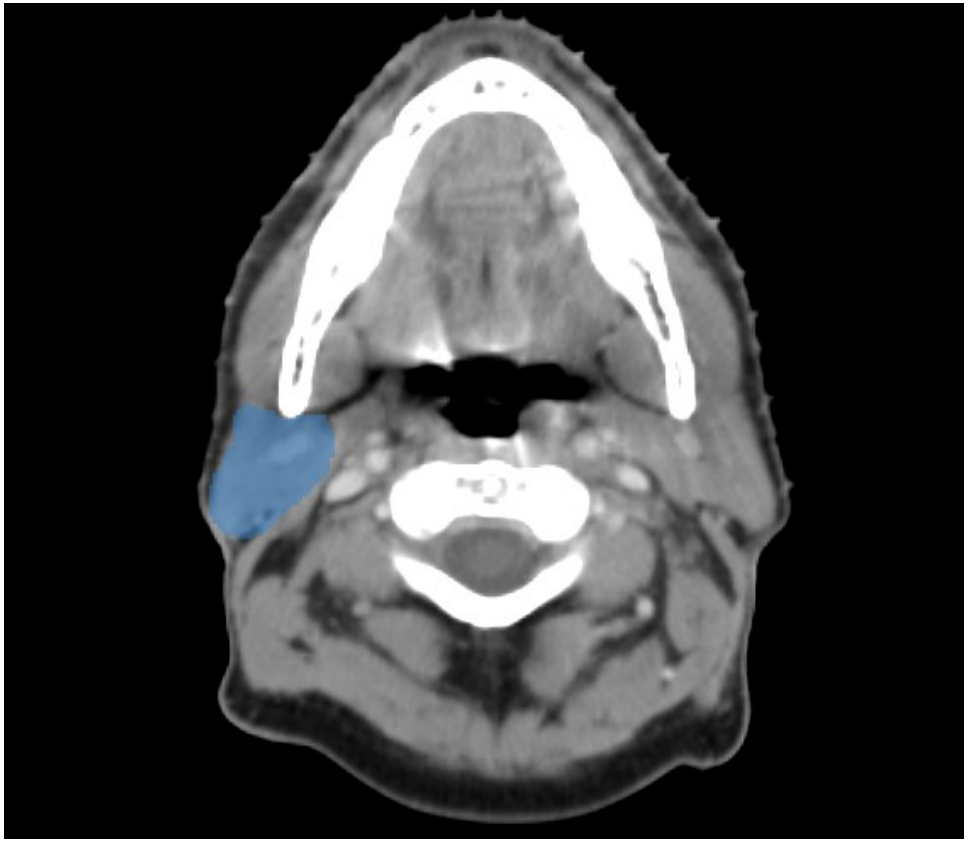

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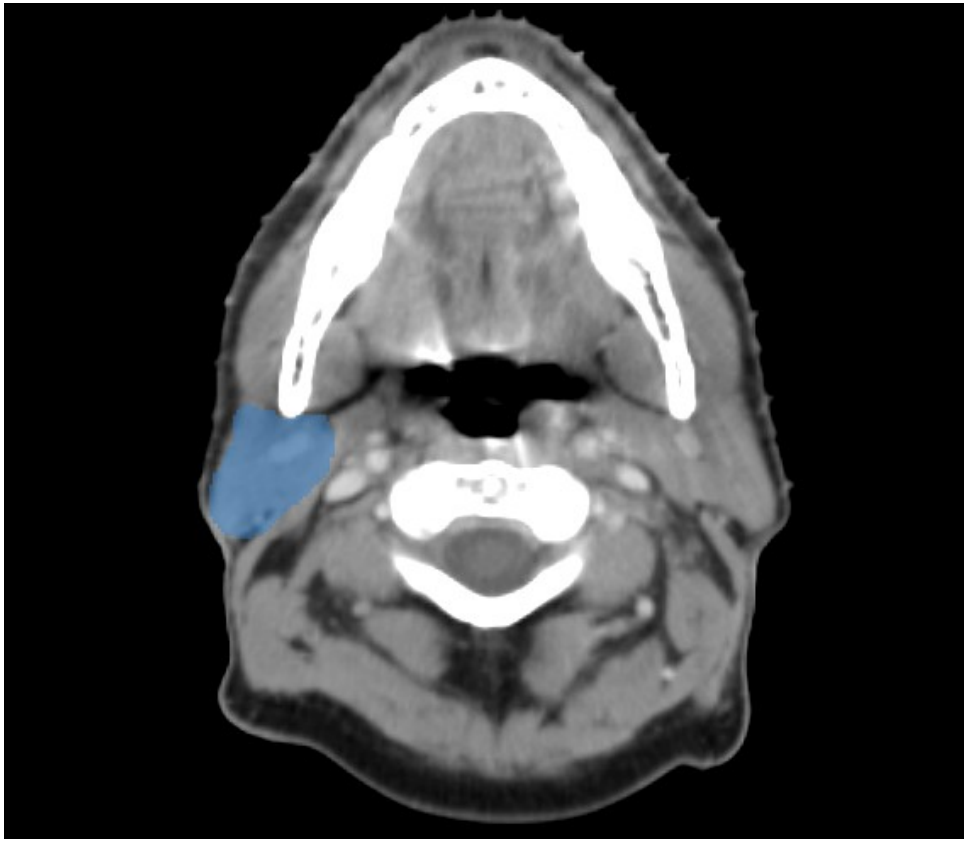
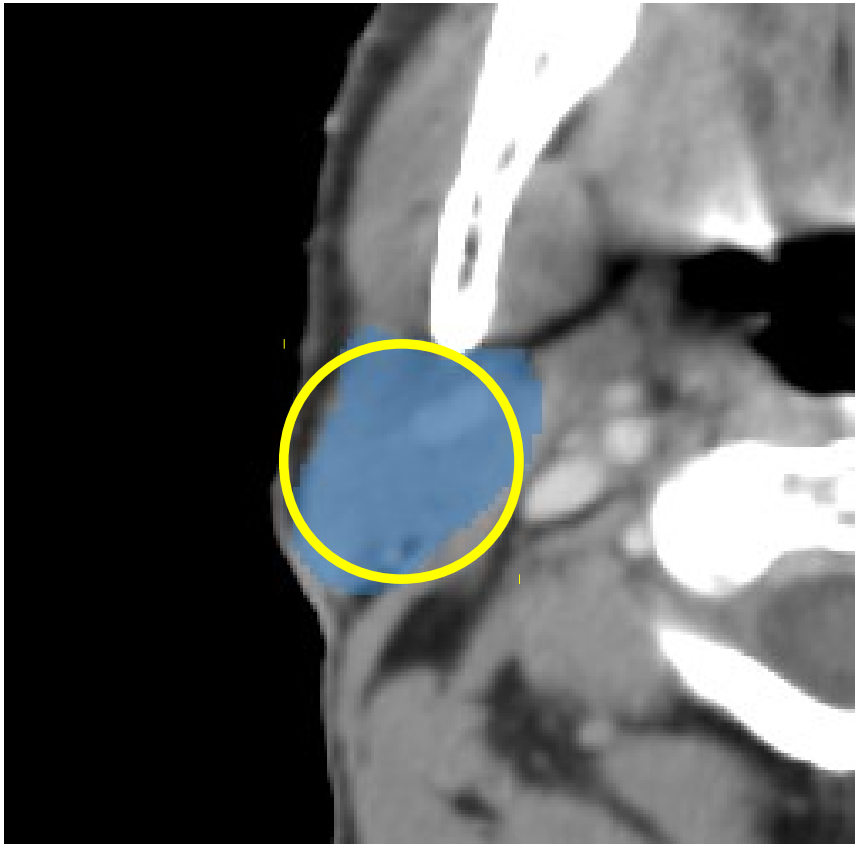
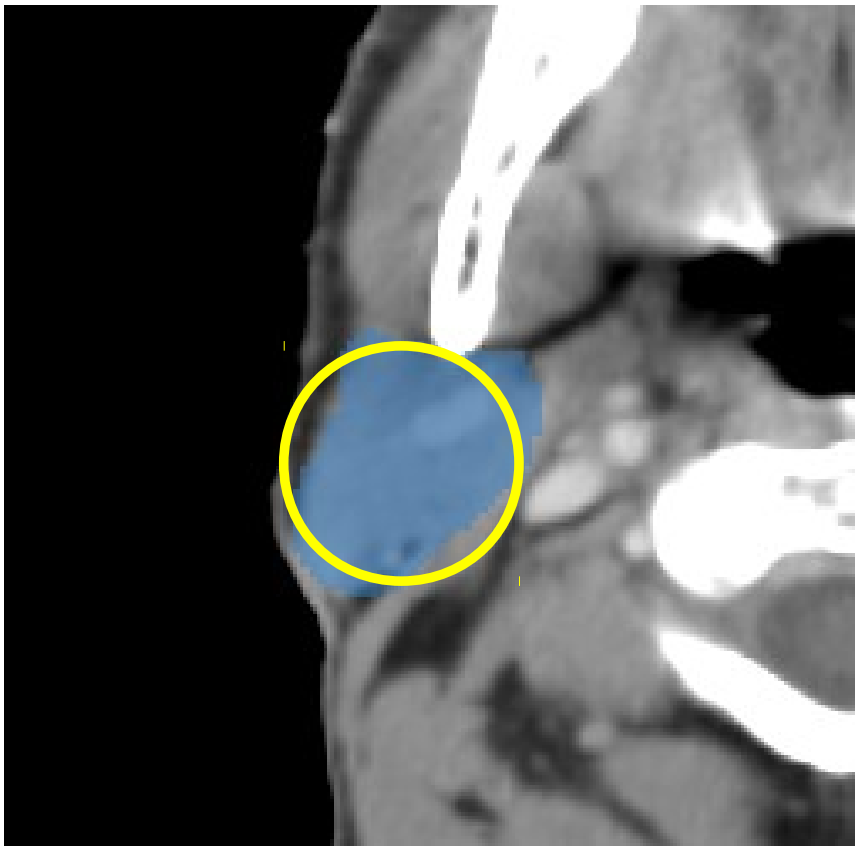




Dice's coefficient

- What is the Dice coefficient when matching a parotid gland with a sphere?

Dice coefficient = 0.74



WARNING

Professional Driver. Closed course.

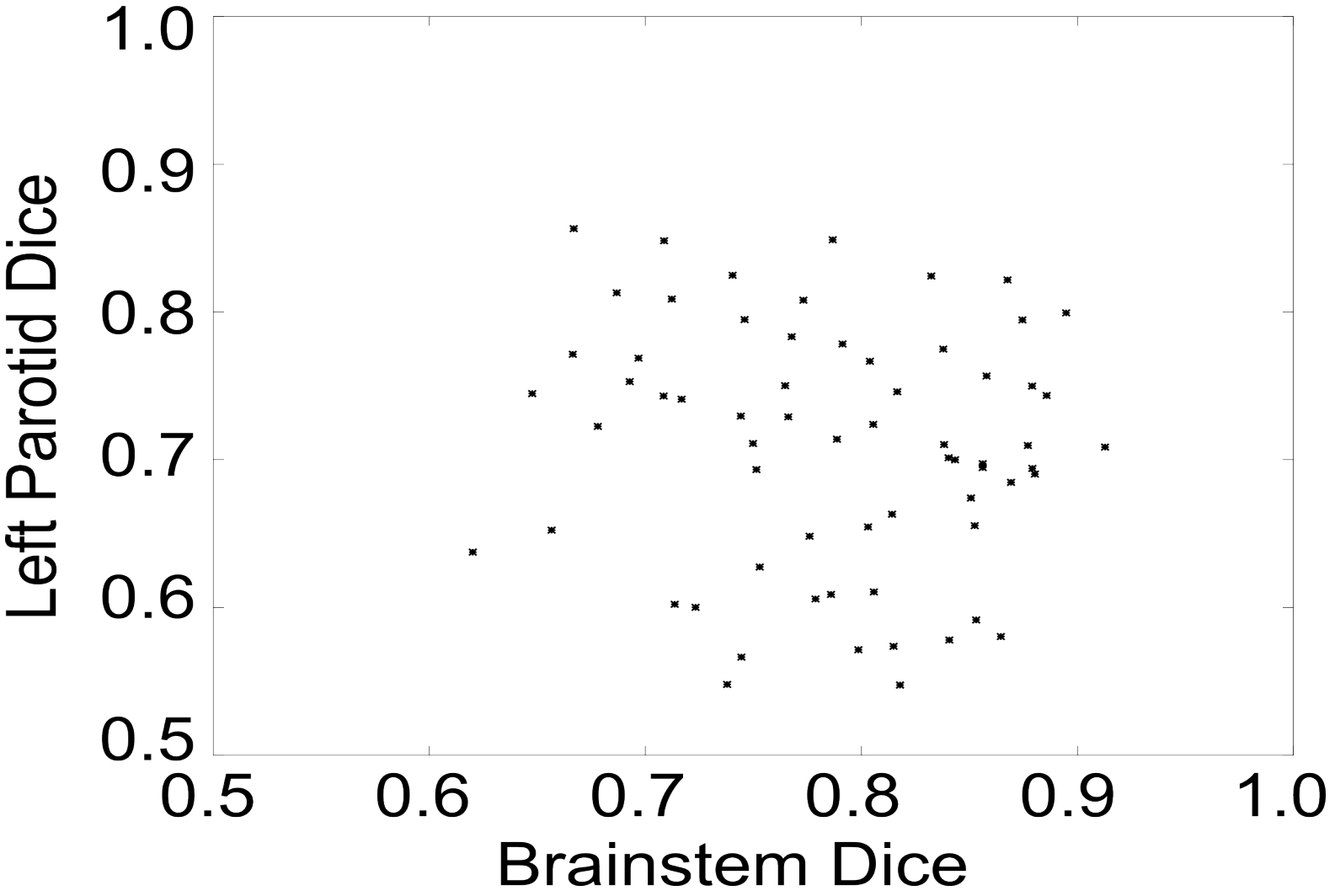
Photo credit: http://nytimes.com

Average Dice

- Average the Dice over all structures of interest

$$\sum_{S \in structures} Dice\left(S, \hat{S}\right)$$

Average Dice



Hausdorff distance

$$d_{\mathrm{H}}(X,Y) = \max\{ \sup_{x\in X} \inf_{y\in Y} d(x,y), \sup_{y\in Y} \inf_{x\in X} d(x,y) \}$$

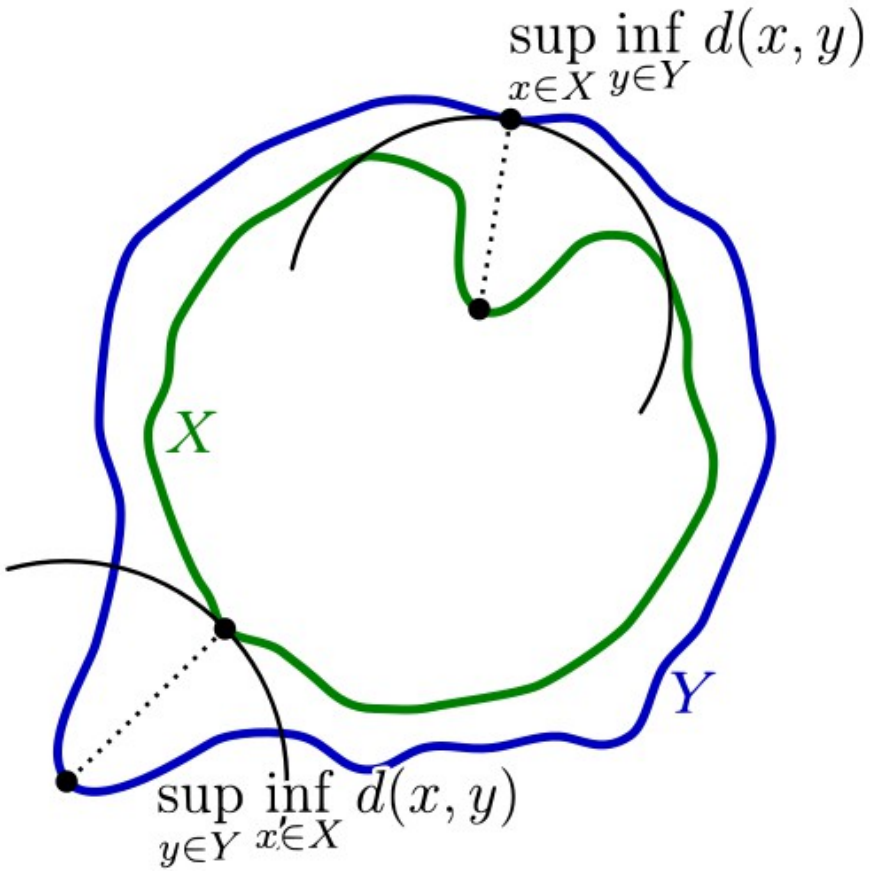


Image credit: http://wikipedia.org

Hausdorff distance

- One-sided Hausdorff distance

$$Hausdorff_{1}(A, B) = \max_{a \in A} \min_{b \in B} |a - b|$$

- Average Hausdorff (take one)

$$\begin{aligned} Hausdorff_{Ave}(A, B) &= \frac{1}{2} Hausdorff_{1}(A, B) \\ &+ \frac{1}{2} Hausdorff_{1}(B, A) \end{aligned}$$

Hausdorff distance

- One-sided Average distance

$$Hausdorff_{1,Ave}(A,B)=\frac{1}{|A|}\sum_{a\in A}\min_{b\in B}|a-b|$$

- Average Hausdorff (take two)

$$Hausdorff_{Ave}(A,B)=max\begin{pmatrix}Hausdorff_{1,Ave}(A,B)\\Hausdorff_{1,Ave}(B,A)\end{pmatrix}$$

- Or is it the average of the two instead of max??

Hausdorff distance

- One-sided fractional (95%) Hausdorff

$$Hausdorff_{1,95}(A, B) = P_{\substack{95 \\ a \in A}}\left(\min_{b \in B} |a - b| \right)$$

Hausdorff distance

- One-sided fractional (95%) Hausdorff

$$Hausdorff_{1,95}(A, B) = P_{\substack{95 \\ a \in A}}\left(\min_{b \in B} |a - b| \right)$$

- Fractional (95%) Hausdorff

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Hausdorff distance

- One-sided fractional (95%) Hausdorff

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- Fractional (95%) Hausdorff

$$Hausdorff_{95}(A, B) = max\begin{pmatrix} Hausdorff_{1,95}(A, B) \\ Hausdorff_{1,95}(B, A) \end{pmatrix}$$

- Or is it the average?
- Or should I combine the points, then take 95%?

Here be dragons

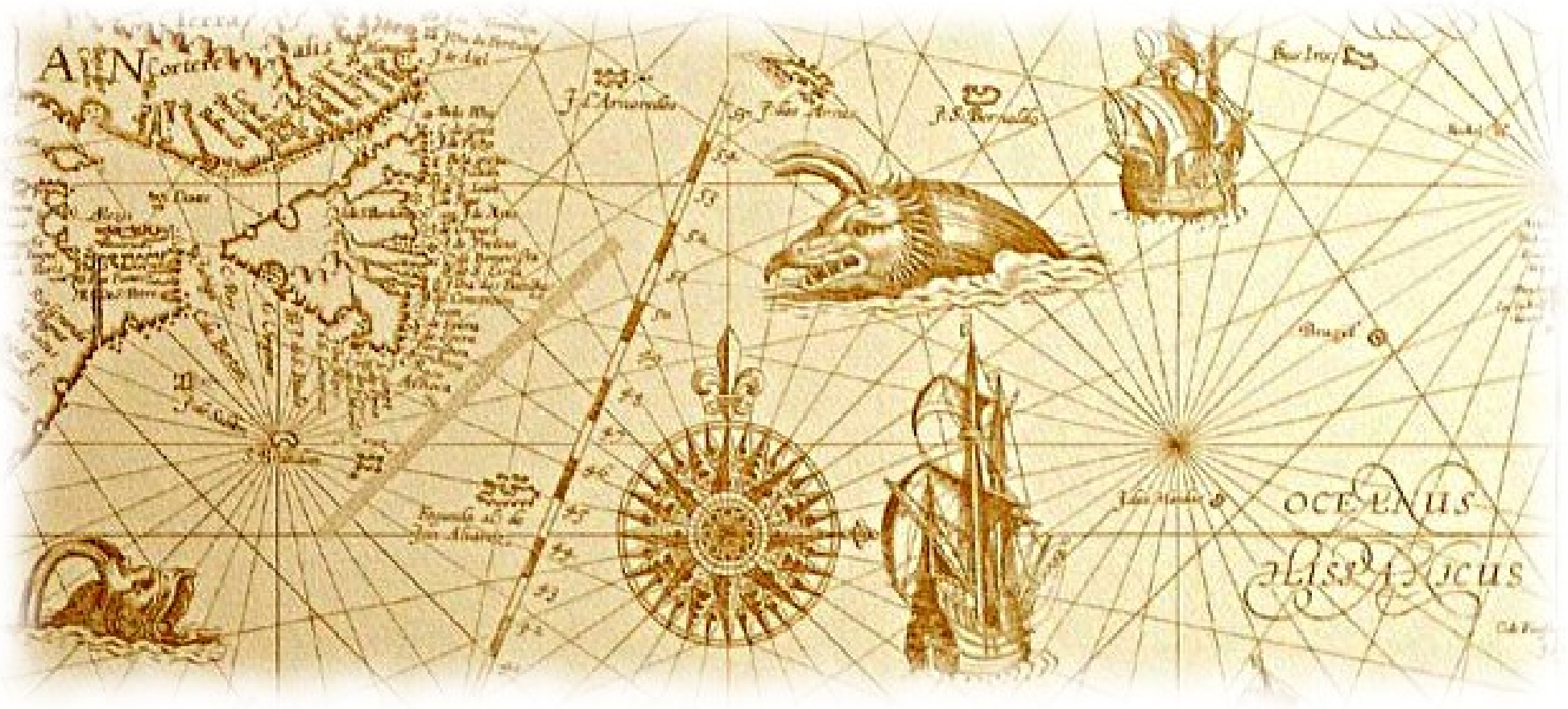
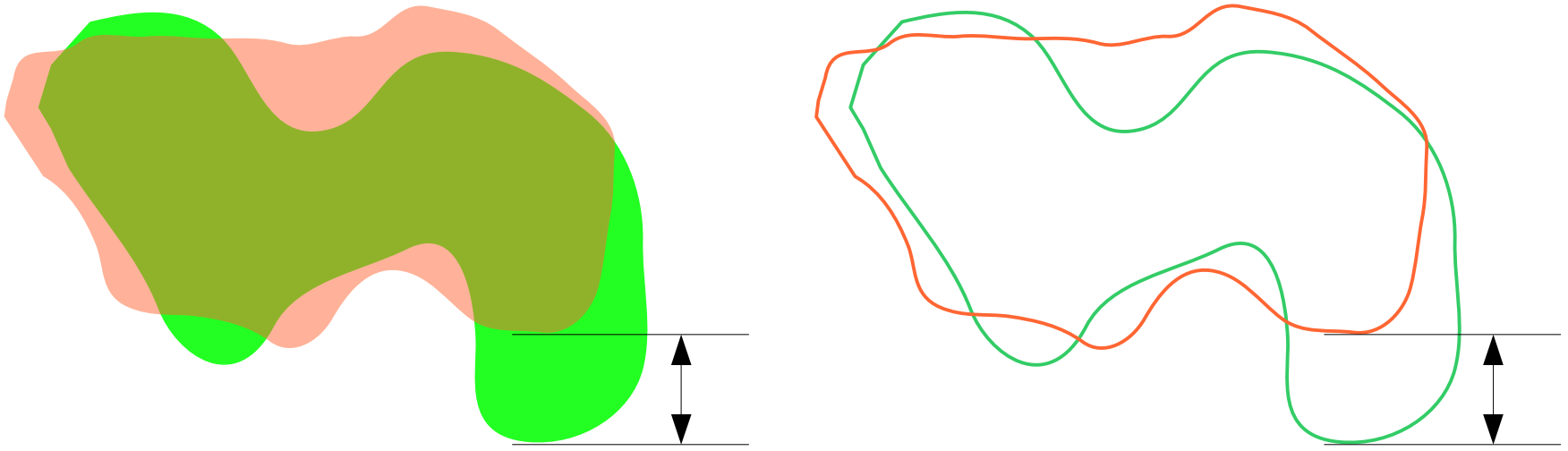


Photo credit: Public Domain

Boundary Hausdorff

- Hausdorff distance may be computed on the *set* or the *set boundary*



Boundary Hausdorff

- The max distance is to a point on the boundary
- So no difference, right?

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- Average distance will change

$$Hausdorff_{Ave}(\partial A, \partial B) = \cdots$$

- 95% distance will change

$$Hausdorff_{95}(\partial A, \partial B) = \cdots$$

Boundary Hausdorff

- "max" Hausdorff changes too

$$Hausdorff\left(A, B\right) \neq Hausdorff\left(\partial A, \partial B\right)$$

Boundary Hausdorff

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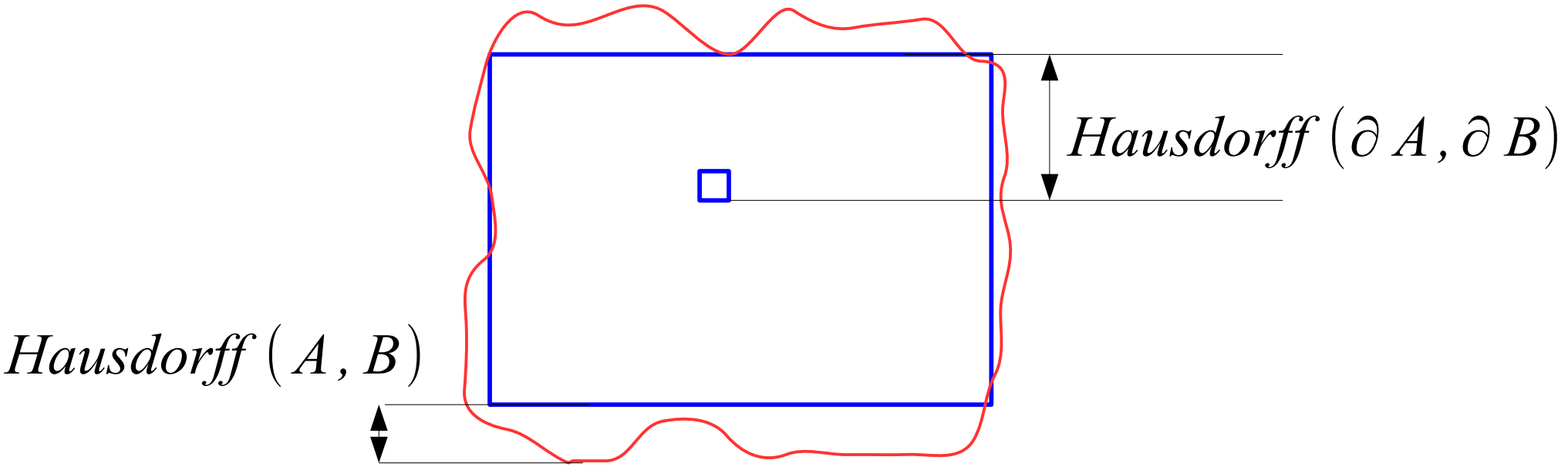

Don't try this at home!

Photo credit: http://missvickie.com

Di-HaRD

Dice – **Ha**usdorff **R**evolutionary **D**istance measure

$$
\begin{aligned}
DiHaRD(A,B) \quad &= \quad \alpha\, Dice(A,B) \\
&+ \quad \beta\, Hausdorff\,(A,B) \\
&+ \quad \gamma\, Hausdorff\,(\partial A, \partial B) \\
&+ \quad \theta\, Hausdorff_{95}(A,B) \\
&+ \quad \nu\, Hausdorff_{95}(\partial A, \partial B) \\
&+ \quad \cdots
\end{aligned}
$$