



NA-MIC

*National Alliance for Medical Image Computing*

*<http://na-mic.org>*

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# Diffusion Tensor Processing and Visualization

Guido Gerig

University of Utah

NAMIC: National Alliance for  
Medical Image Computing

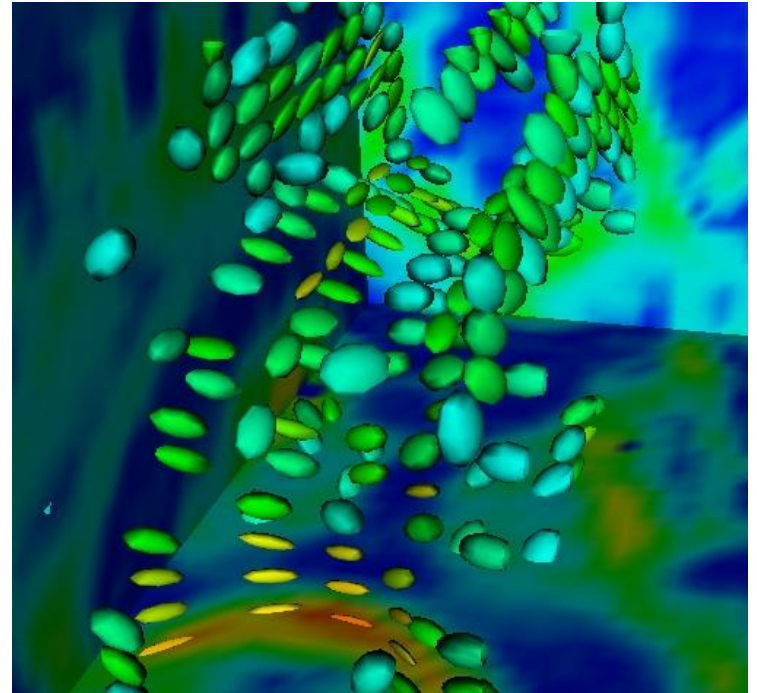




# Acknowledgments

## Contributors:

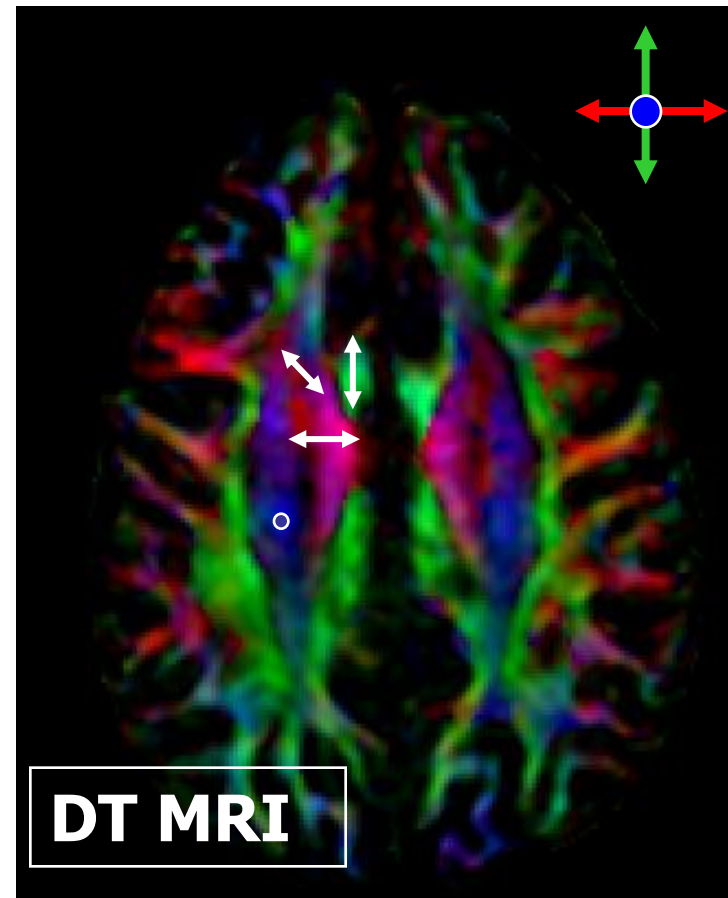
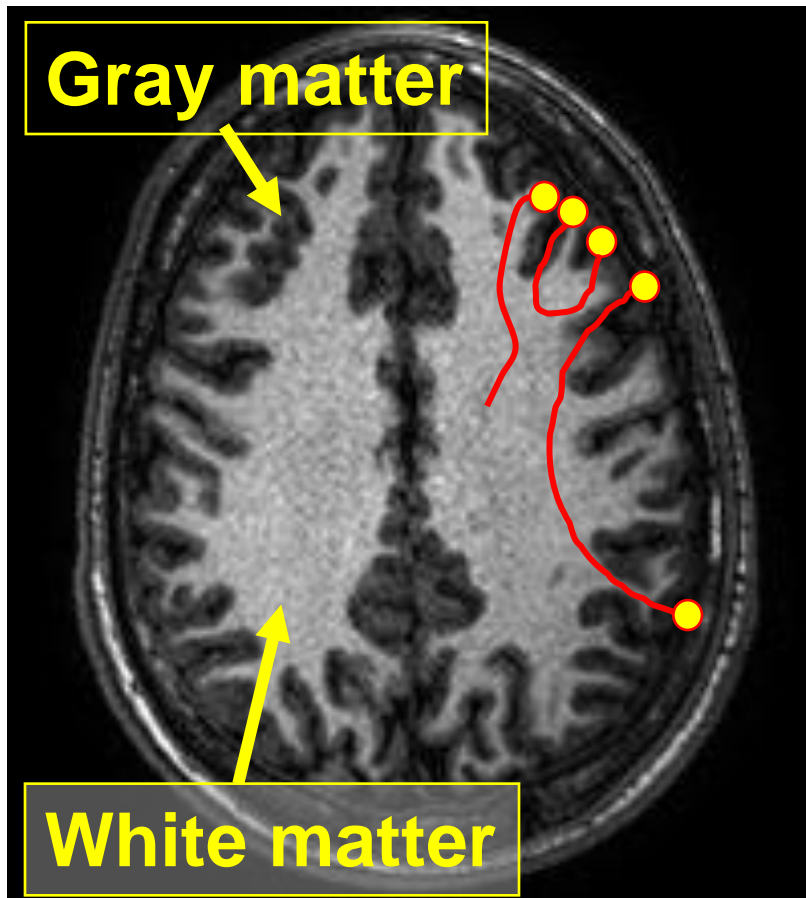
- C-F. Westin
- A. Alexander
- G. Kindlmann
- L. O'Donnell
- C. Goodlett
- J. Fallon
- R. Whitaker
- R. McKinstry



National Alliance for Medical Image Computing (NIH U54EB005149)



# Diffusion Tensor Imaging (DT MRI) reveals White Matter Structure

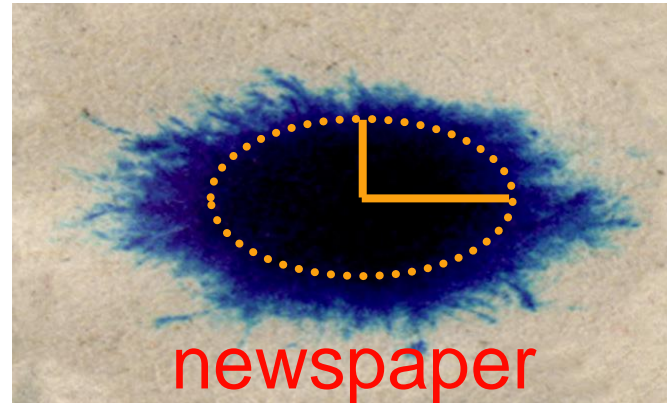


Courtesy of Susumu Mori, JHU



# Diffusion in Biological Tissue

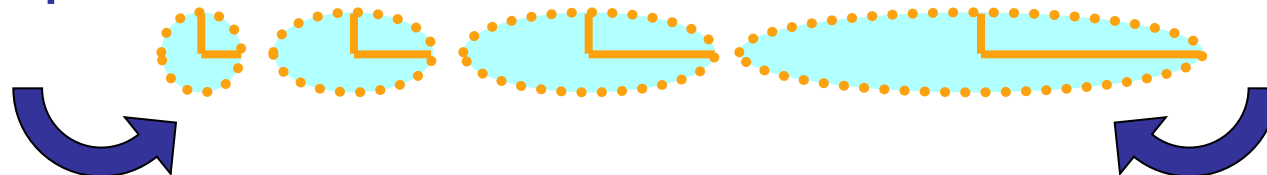
- Motion of water through tissue
- Sometimes faster in some directions than others



- Anisotropy: diffusion rate depends on direction

isotropic

anisotropic

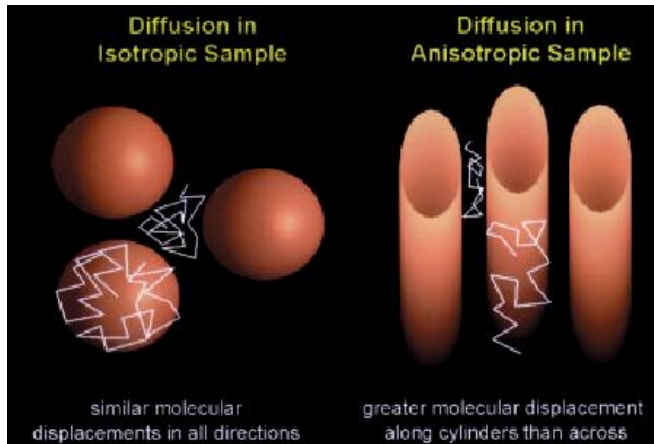


G. Kindlmann



# Diffusion in White Matter

- Diffusion of water molecules



From Beaulieu[02]

- Reflects the underlying structure of the tissues
  - Faster diffusion along fibers than perpendicular to them
  - Water diffusion anisotropy used to track fibers, estimate white matter integrity
- Tensor model [Basser]
  - Determine the whole tensor to estimate diffusion anisotropy



# The Physics of Diffusion

- Density of substance changes (evolves) over time according to a differential equation (PDE)

$$\frac{\partial \mu}{\partial t} = \nabla \cdot D \nabla \mu$$

Change in  
density

Diffusion – matrix,  
tensor  
(2x2 or 3x3)

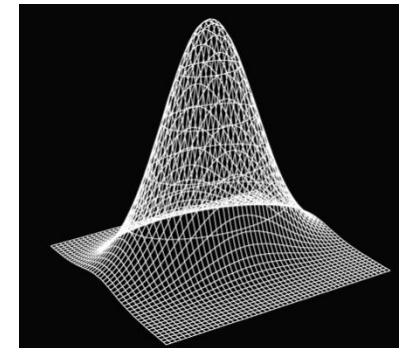
Derivatives  
(gradients) in  
space



# Solutions of the Diffusion Equation

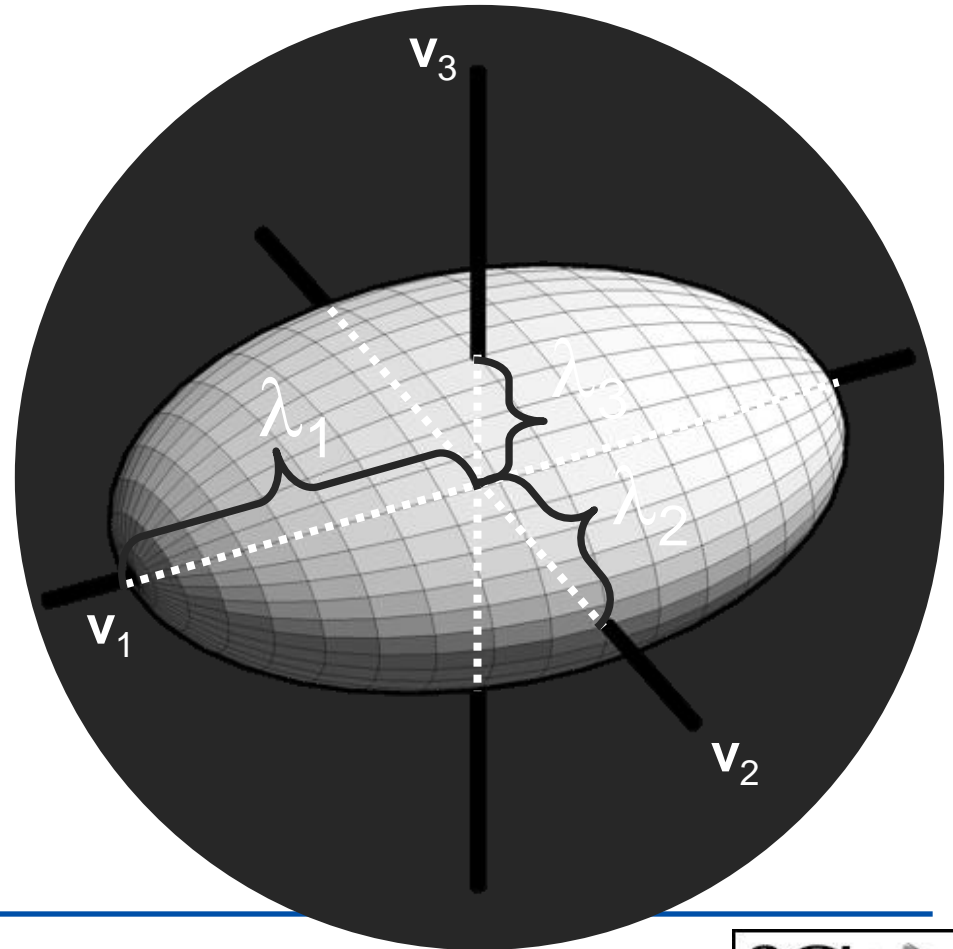
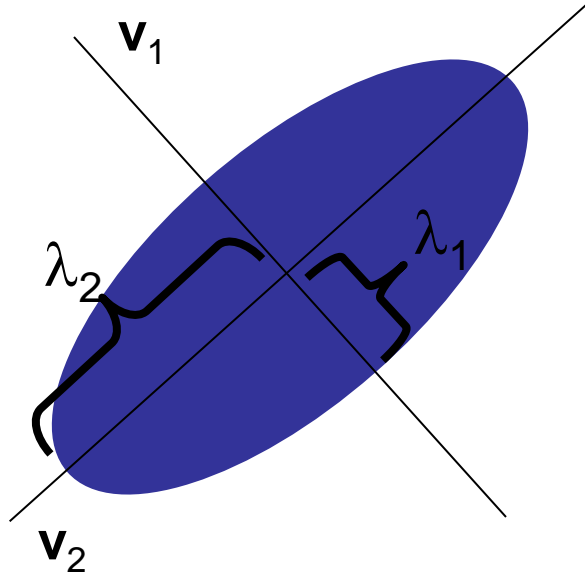
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- Simple assumptions
  - Small dot of a substance (point)
  - $D$  constant everywhere in space
- Solution is a multivariate Gaussian
  - Normal distribution
  - “ $D$ ” plays the role of the covariance matrix
- This relationship is not a coincidence
  - Probabilistic models of diffusion (random walk)





# Eigen Directions and Values (Principle Directions)







# Tensors From Diffusion-Weighted Images

- Big assumption
  - At the scale of DW-MRI measurements
  - Diffusion of water in tissue is approximated by Gaussian
    - Solution to heat equation with constant diffusion tensor
- **Stejskal-Tanner equation**
  - Relationship between the DW images and tensor D

$$S_k = S_0 e^{-bg_k^T D g_k}$$

$k^{\text{th}}$  DW Image

Base image

Gradient direction

Physical constants  
Strength of gradient  
Duration of gradient pulse  
Read-out time

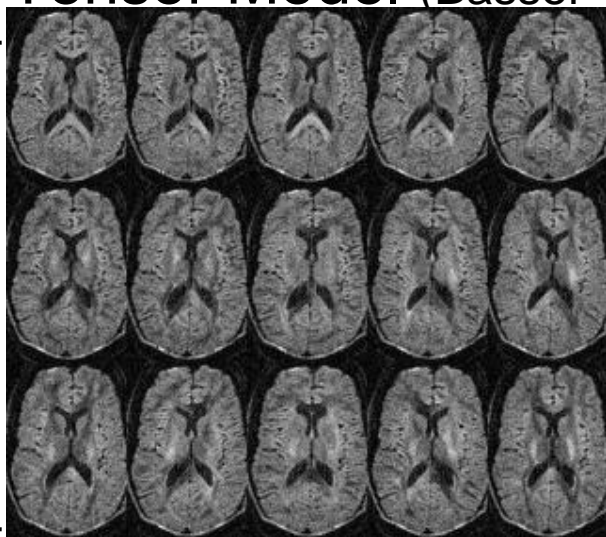


# DWI summary: Model

Single Tensor Model (Basser 1994)

$$A_i = A_0 e^{-b g_i^T D g_i}$$

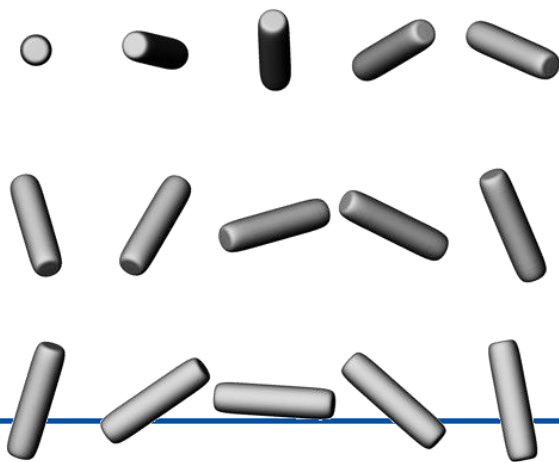
$A_i$



$A_0$

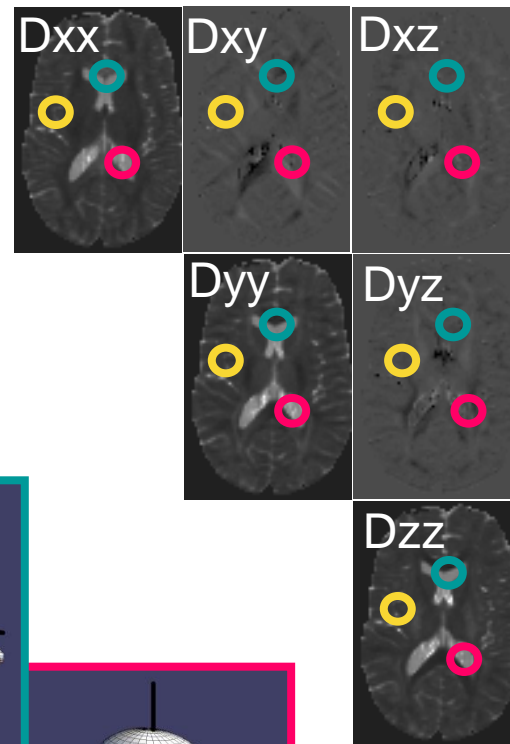
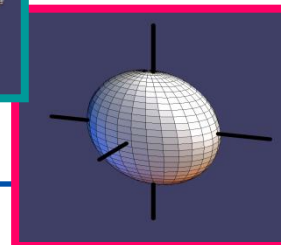
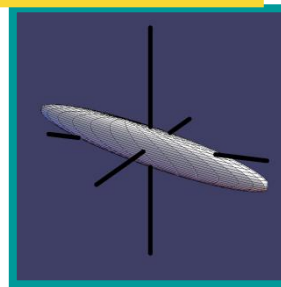
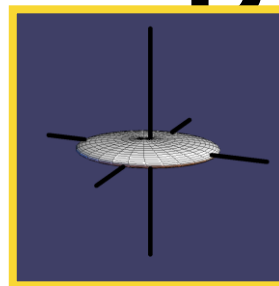


$g_i$



Tensor estimation

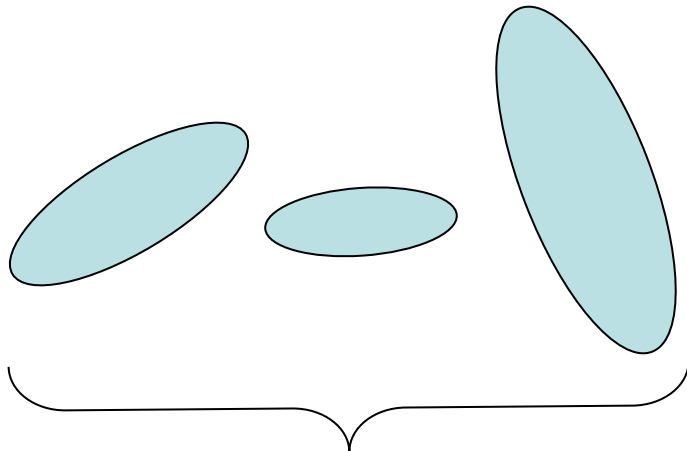
**D**



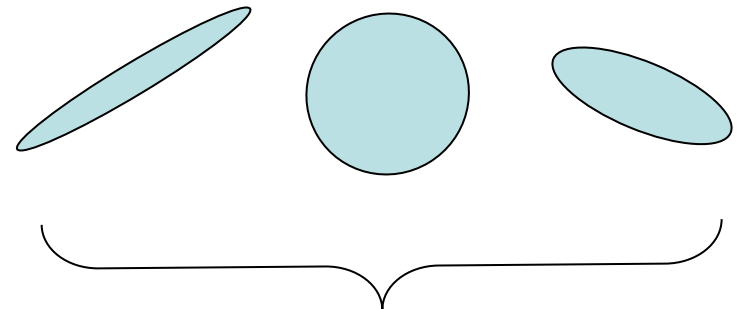


# Shape Measures on Tensors

- Represent or visualization shape
- Quantify meaningful aspect of shape
- Shape vs size



Different sizes/orientations



Different shapes



# Measuring the “Size” of a Tensor

- Length –  $(\lambda_1 + \lambda_2 + \lambda_3)/3$   
–  $(\lambda_1^2 + \lambda_2^2 + \lambda_3^2)^{1/2}$
- Area –  $(\lambda_1 \lambda_2 + \lambda_1 \lambda_3 + \lambda_2 \lambda_3)$
- Volume –  $(\lambda_1 \lambda_2 \lambda_3)$

Sometimes used.

Also called:

“Root sum of squares”

“Diffusion norm”

“Frobenius norm”

Generally used.

Also called:

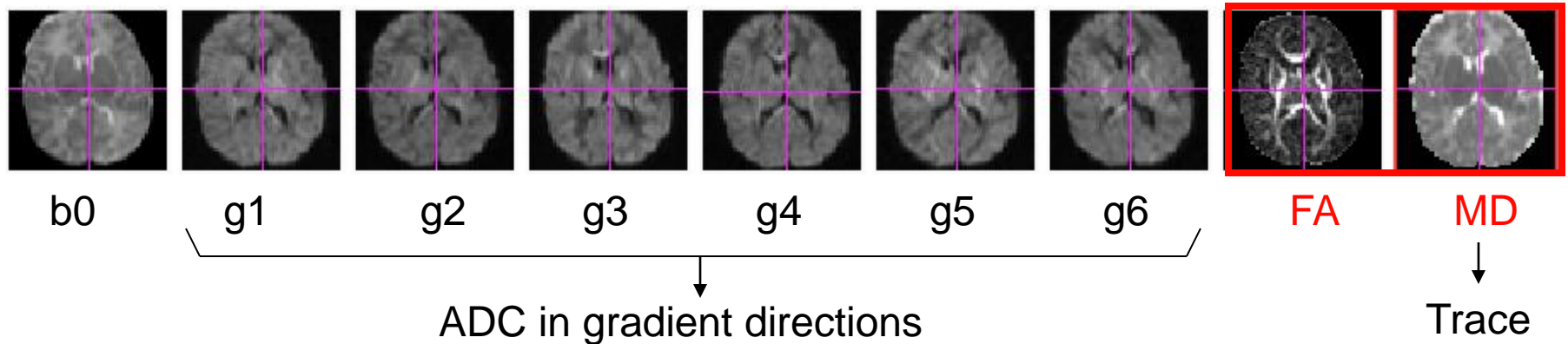
“Mean diffusivity”  $\langle MD \rangle$

“Trace”



# ADC versus Mean Diffusivity

- Apparent diffusion coefficient (ADC) measures diffusivity in a specific direction.
- Mean diffusivity ( $\langle MD \rangle$ ) is the trace of the diffusion tensor.
- Terms often not properly used, papers often cite ADC but actually mean  $\langle MD \rangle$



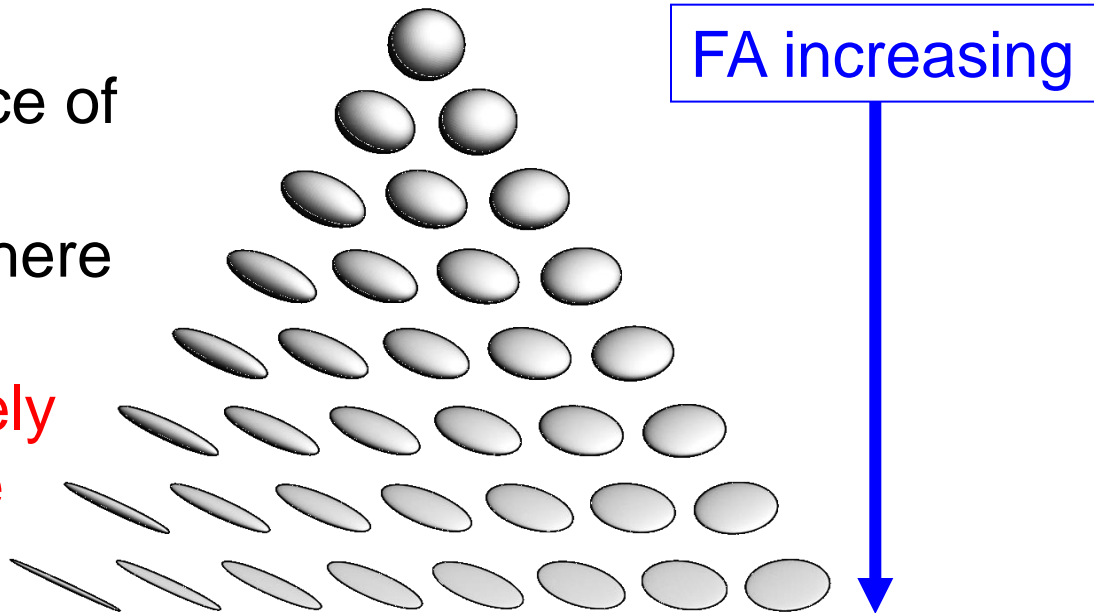


# Reducing Shape to One Number Fractional Anisotropy

$$FA = \frac{\sqrt{(\lambda_1 - \lambda_2)^2 + (\lambda_1 - \lambda_3)^2 + (\lambda_2 - \lambda_3)^2}}{\sqrt{2}\sqrt{\lambda_1^2 + \lambda_2^2 + \lambda_3^2}}$$

Properties:

- Normalized variance of eigenvalues
- Difference from sphere
- Invariant to size
- **FA does not uniquely characterize shape**

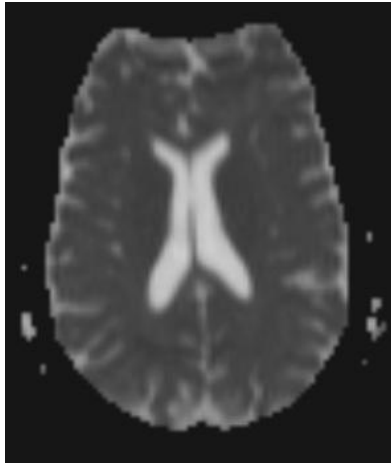




# Tensor size (MD) and shape (FA)

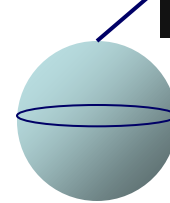
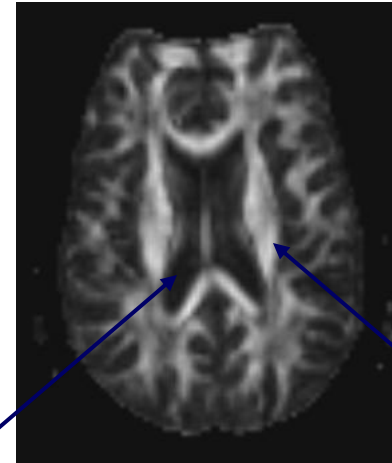
- Mean diffusivity (MD)

$$MD = \frac{\lambda_1 + \lambda_2 + \lambda_3}{3}$$

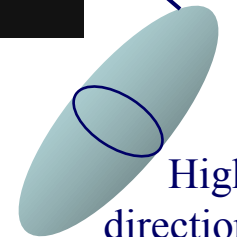


- Fractional anisotropy (FA)

$$FA = \frac{1}{\sqrt{2}} \frac{\sqrt{(\lambda_1 - \lambda_2)^2 + (\lambda_2 - \lambda_3)^2 + (\lambda_1 - \lambda_3)^2}}{\sqrt{\lambda_1^2 + \lambda_2^2 + \lambda_3^2}}$$



Isotropic  
diffusion



Highly  
directional  
diffusion

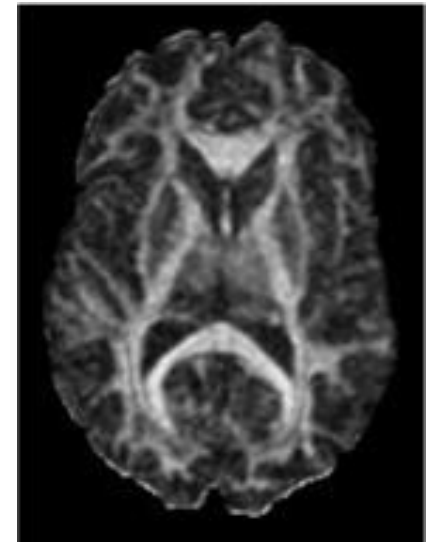




# FA as an Indicator for White Matter

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- Visualization – ignore tissue that is not WM
- Registration – Align WM bundles
- Tractography – terminate tracts as they exit WM
- Analysis
  - Axon density/degeneration/integrity
  - Myelin
- Big question
  - What physiological/anatomical property does FA measure?







# Visualizing Tensors: Direction and Shape

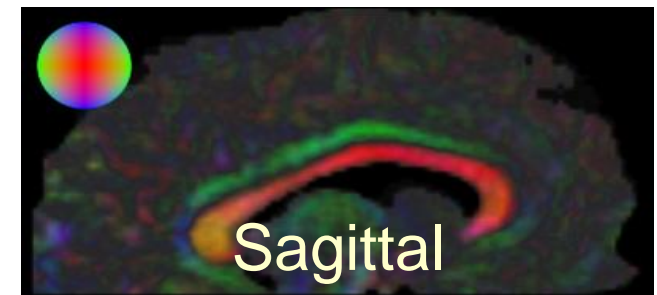
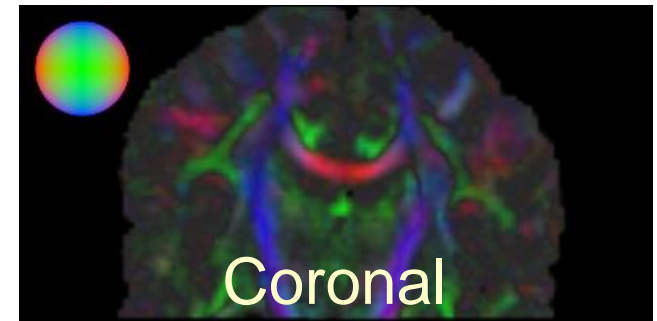
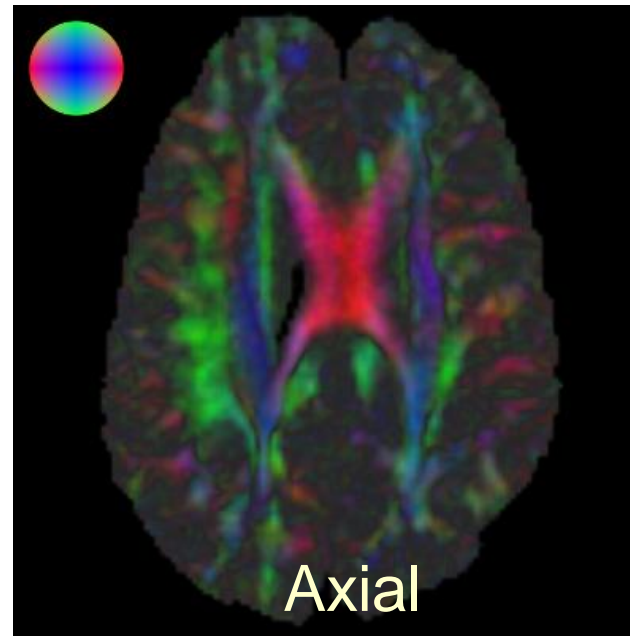
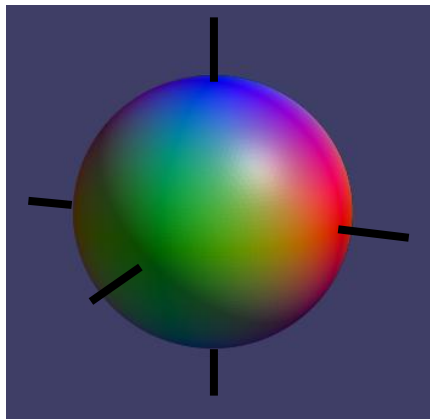
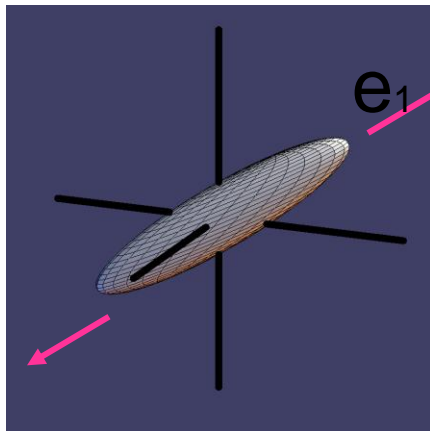
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- Color mapping
- Glyphs



# Coloring by Principal Diffusion Direction

- Principal eigenvector, linear anisotropy determine color



$$R = | \mathbf{e}_1 \cdot \mathbf{x} |$$

$$G = | \mathbf{e}_1 \cdot \mathbf{y} |$$

$$B = | \mathbf{e}_1 \cdot \mathbf{z} |$$

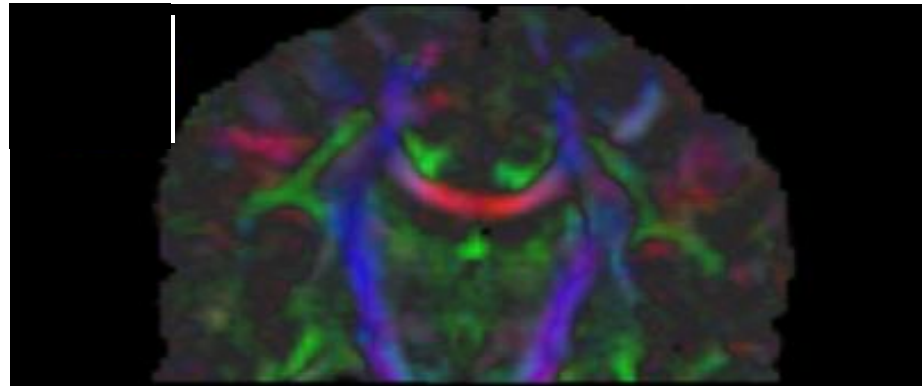
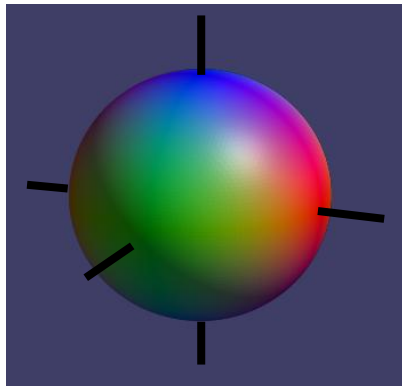
C. Pierpaoli, 1997

Slide G. Kindlmann



# Issues With Coloring by Direction

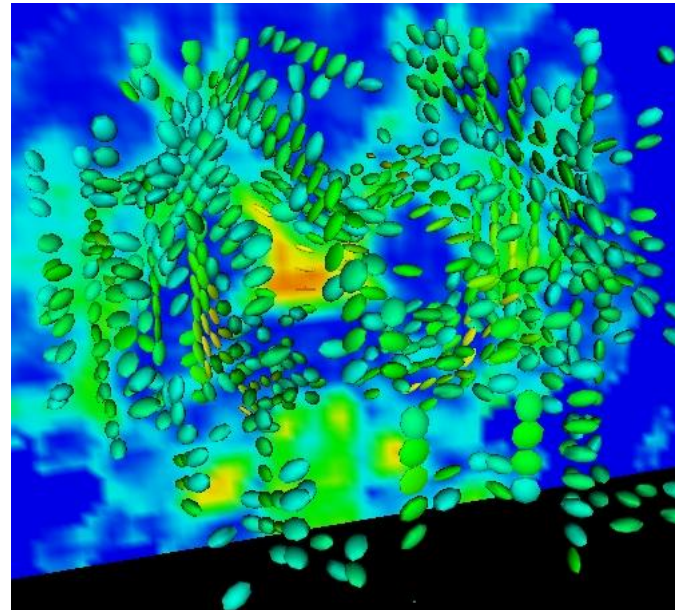
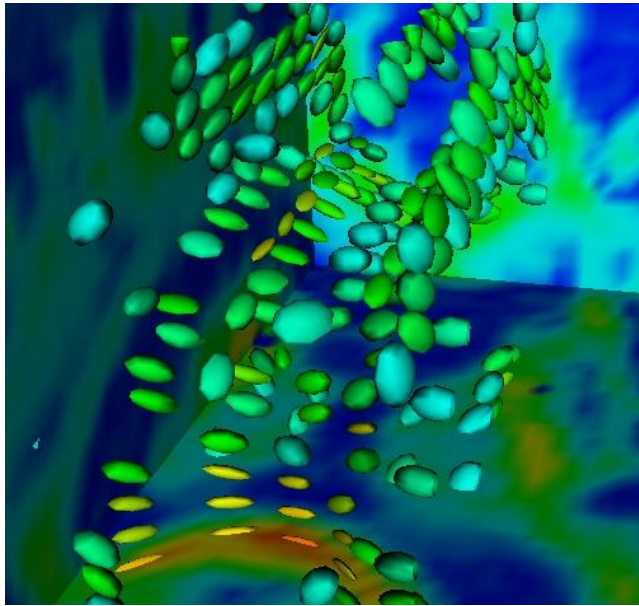
- Set transparency according to FA (highlight-tracts)
- Coordinate system dependent
- Primary colors dominate
  - Perception: saturated colors tend to look more intense
  - Which direction is “cyan”?
  - Coloring is not unique



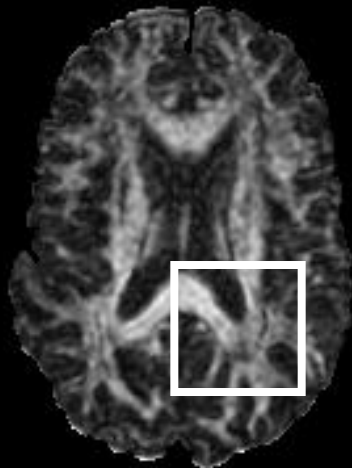


# Visualization with Glyphs

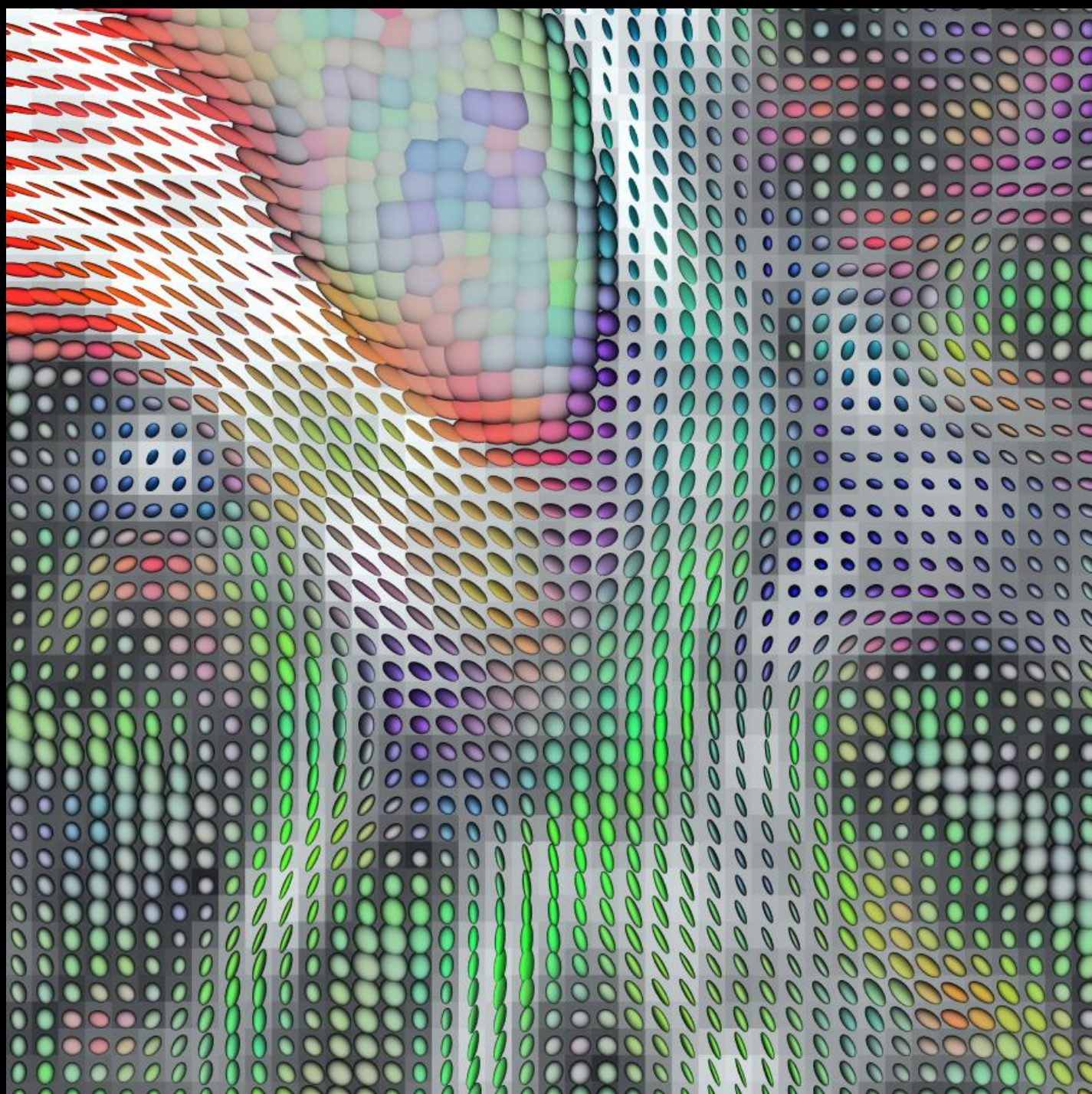
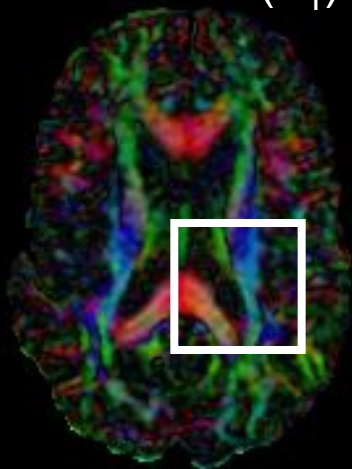
- Density and placement based on FA or detected features
- Place ellipsoids on regular grid

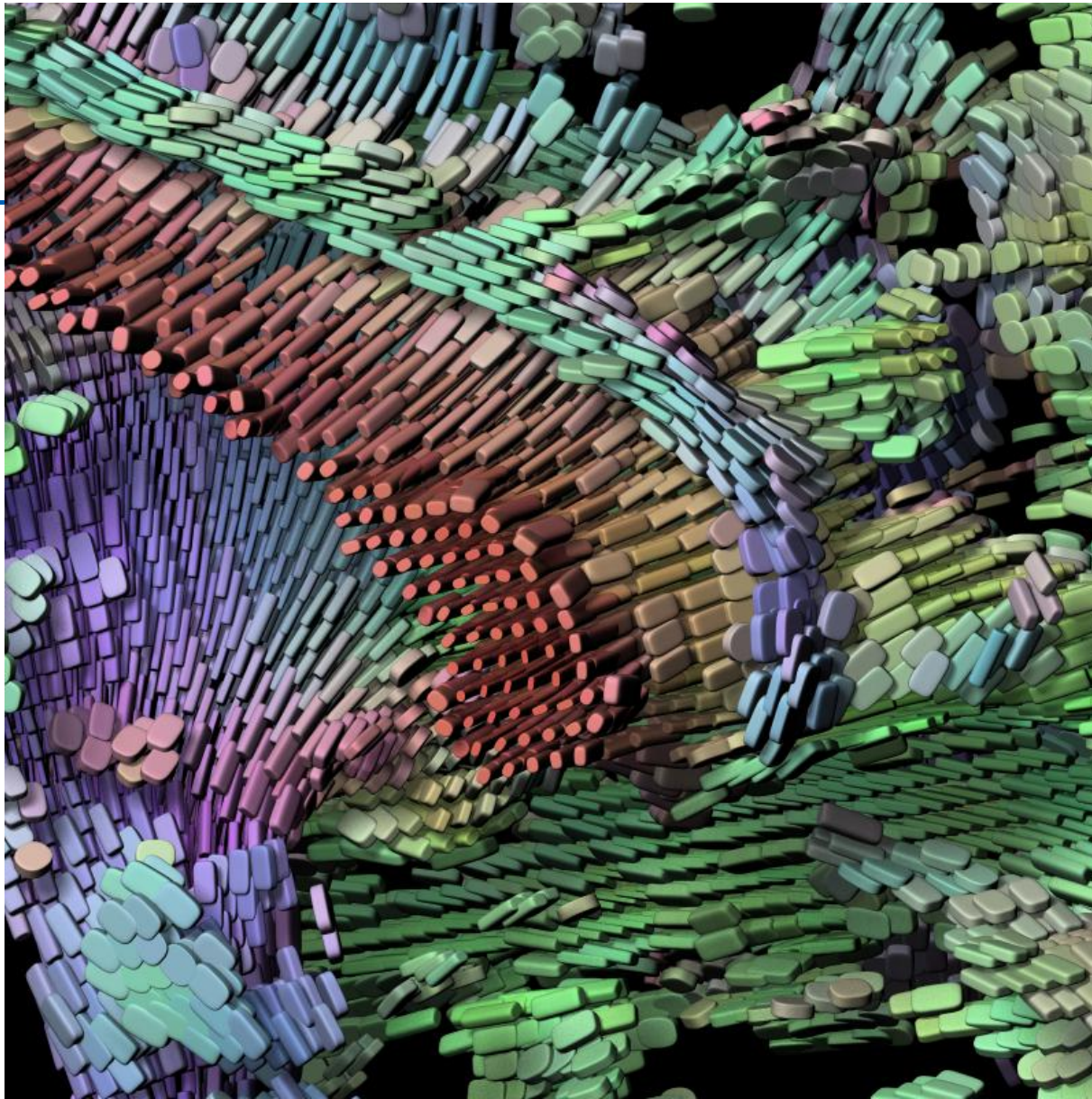


Backdrop: FA



Color: RGB( $\mathbf{e}_1$ )

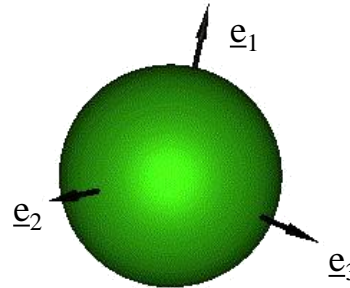
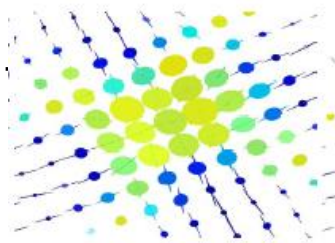




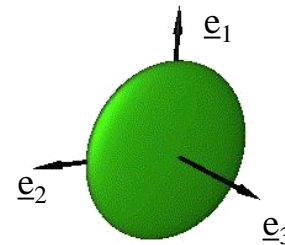


# Prolate vs. oblate: Why do we care?

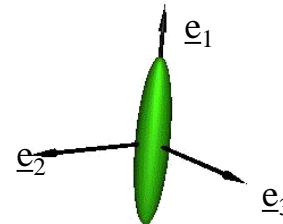
- Free diffusion (ventricles) shown as spheres.
- Intersecting tracts can't be properly modeled by a single tensor: Simplified disks in rank-1 tensors.
- Large tracts can be locally modeled by single tensors.



$\lambda_1 \approx \lambda_2 \approx \lambda_3$  - Isotropic  
Prevalent in CSF and gray matter regions of the brain.



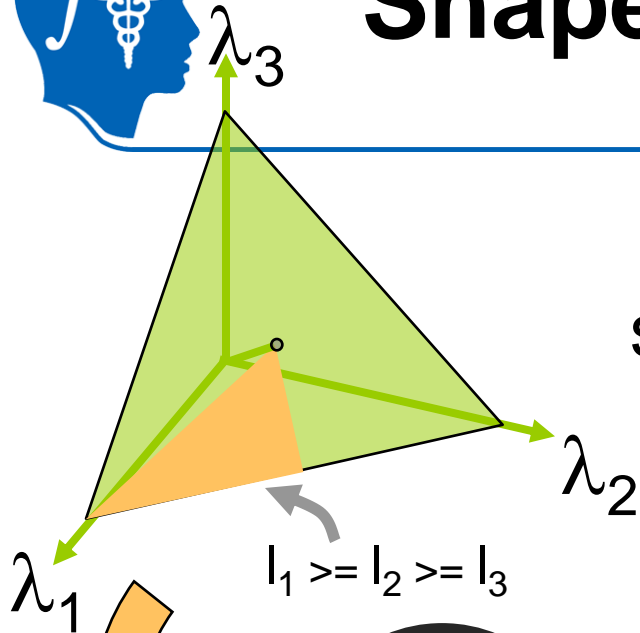
$\lambda_1 \approx \lambda_2 \gg \lambda_3$  - Oblate  
Arise in white matter regions.



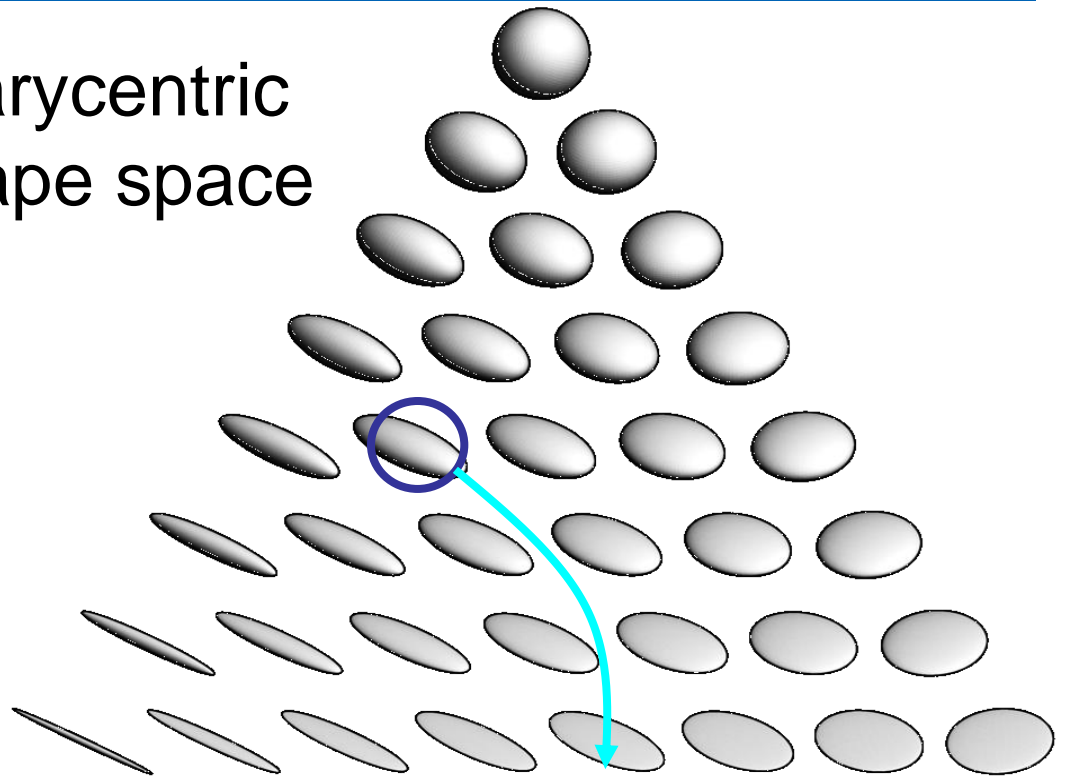
$\lambda_1 \gg \lambda_2 \approx \lambda_3$  - Prolate  
Prevalent in white matter regions.



# Shape Other Than Size

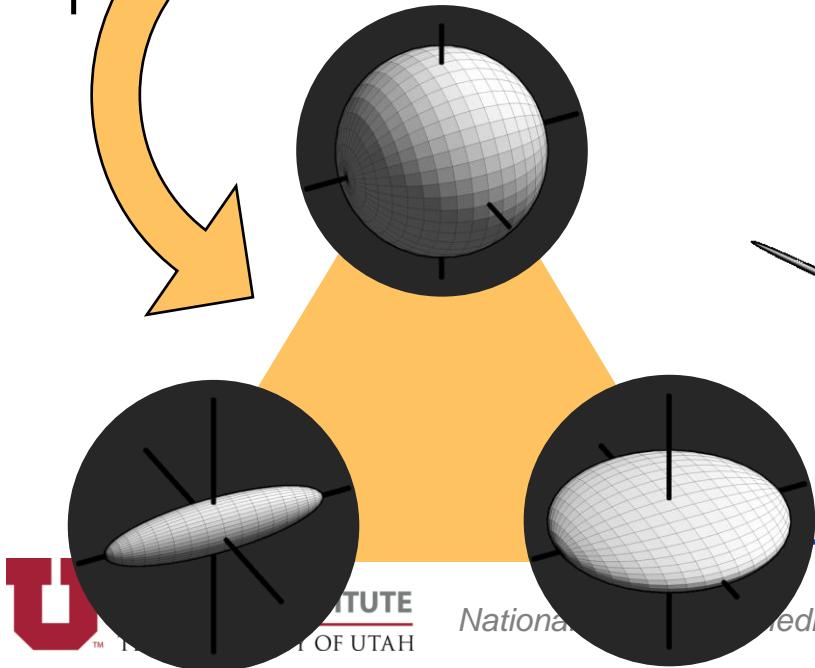


Barycentric  
shape space



$(C_S, C_L, C_P)$

Westin, 1997





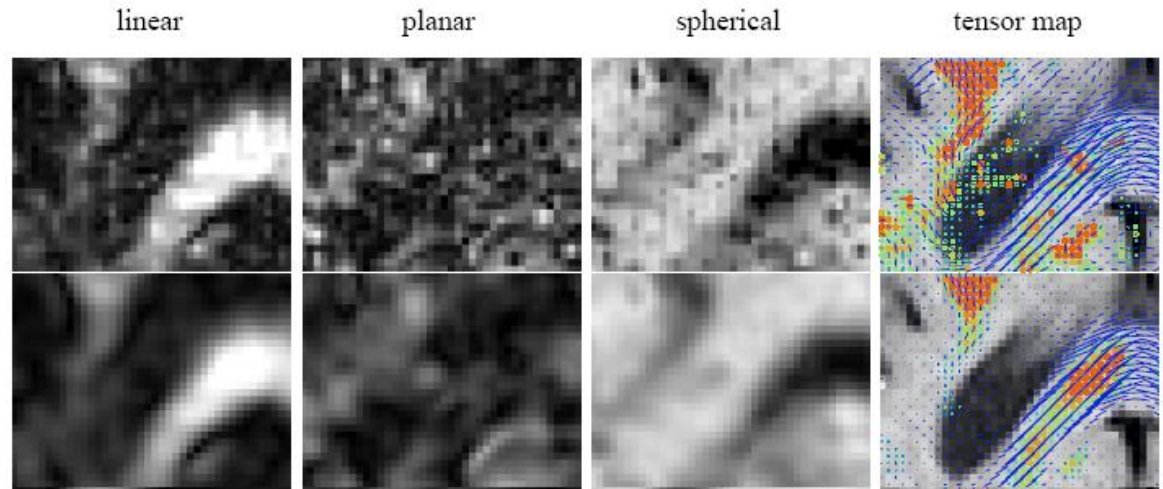


# Shape Characterization: Westin

$$c_l = \frac{\lambda_1 - \lambda_2}{\lambda_1}$$

$$c_p = \frac{\lambda_2 - \lambda_3}{\lambda_1}$$

$$c_s = \frac{\lambda_3}{\lambda_1}$$



$$c_l + c_p + c_s = 1$$

Westin et al., MICCAI'99



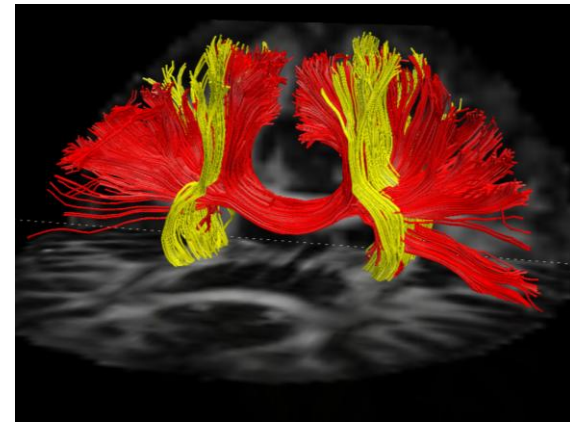
# Dream: Connectivity?



**Forebrain Fiber Bundles: General idea of where various fiber bundles are and regions they interconnect or project to.**



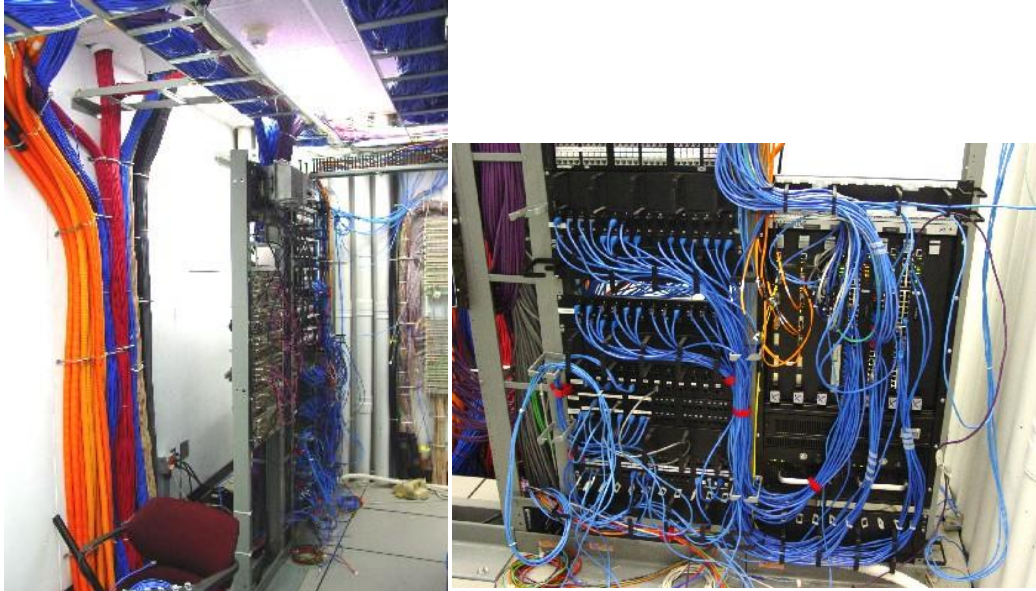
**Source: Duke NeuroAnatomy Web Resources (Ch. Hulette)**



**Tractography: Coronal view (Dell'Acqua et al. NeuroImage '10)**

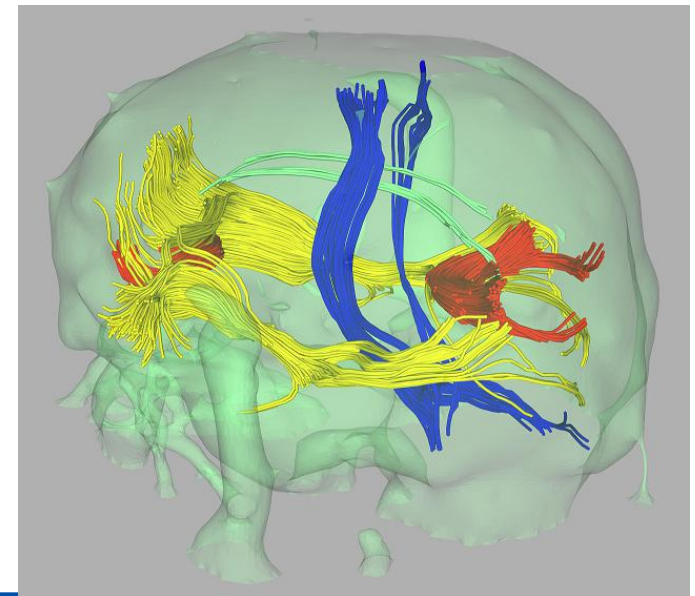


# Networking and Brain Connectivity



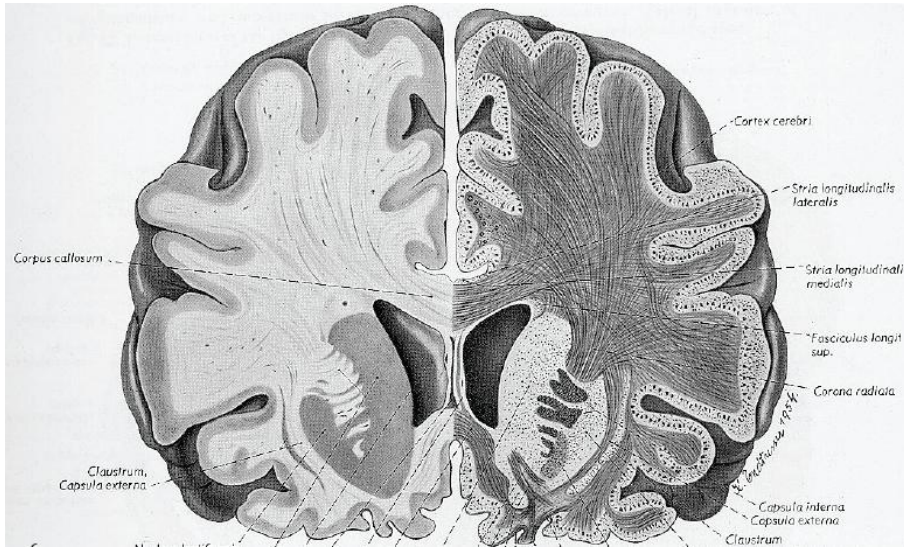
**UNC Computer Science:  
Network wire cabinets**

**Major Fiber  
Tracts extracted  
from DT MRI**

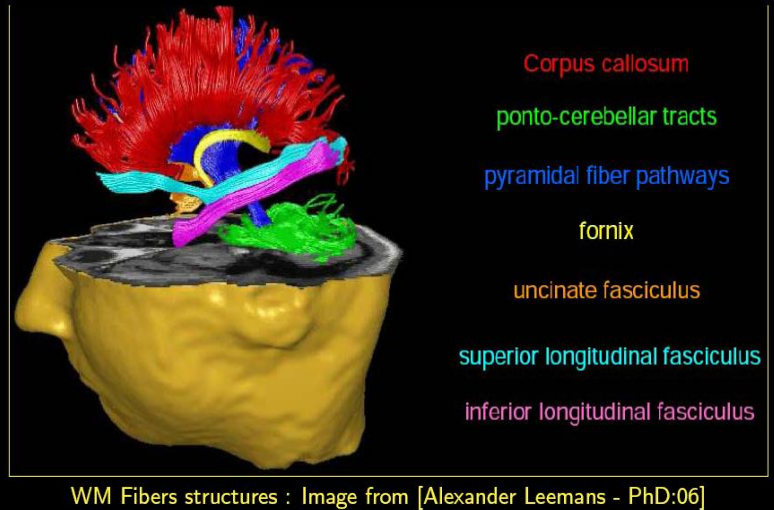




# White Matter Tracts



## WM Fiber Structures of the Human Brain



- In tractography fibers are traced, with the aim to visualize white matter tracts.
- The word “tractography” is not related to “tracking”, but to “tract”.
- White matter tract, white matter fasciculus

Courtesy Carl-Fredrik Westin, MICCAI 2008 workshop



# Fiber Bundles via Tractography

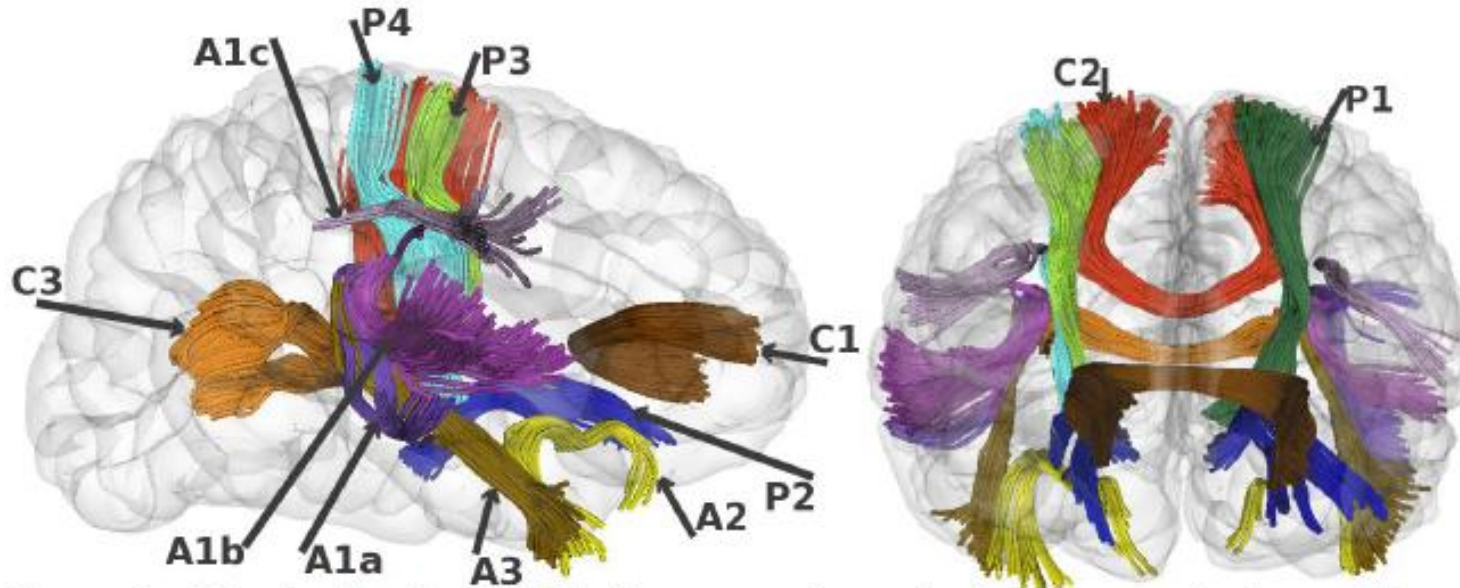


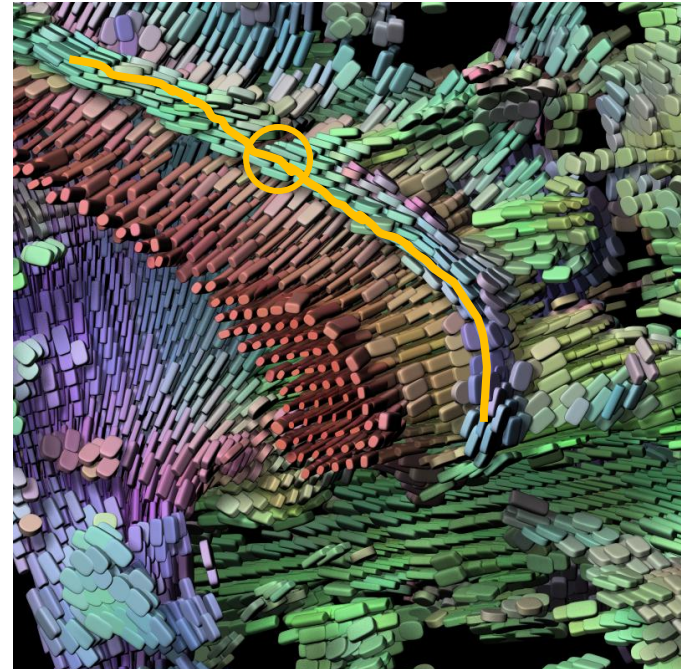
Figure 2. 3D visualization of 21 fiber tracts in sagittal and coronal views. C1, genu corpus callosum (CC); C2, body CC; C3, splenium CC. A1a, arcuate-inferior-temporal tract; A1b, arcuate-superior-temporal tract; A1c, arcuate-superior tract; A2, uncinate fasciculus; A3, inferior longitudinal fasciculus (ILF); P1, posterior limb internal capsule (PLIC); P2, anterior limb internal capsule (ALIC); P3, motor tract; P4 sensory tract.

Geng, Gilmore, Gerig et al., submitted



# From Tensors to Connectivity?

- Study diffusivity in 3D tensor field
- Propagate principal diffusion direction originating at user-selected seed point
- Display paths as streamlines
- Measurement of FA and MD along path



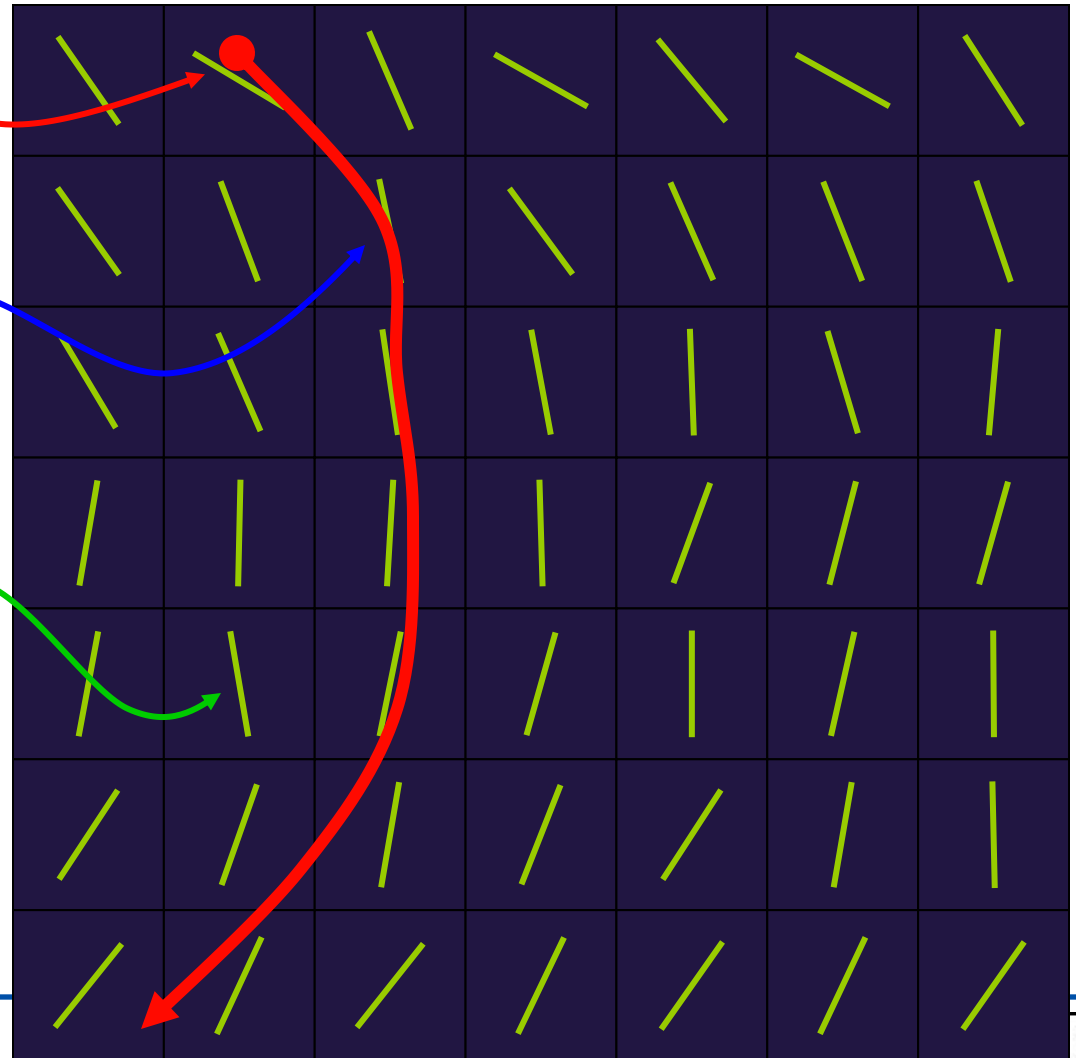


# DTI Tractography

Seed point(s)

Move marker in discrete steps and find next direction

Direction of principle eigen value





# Going Beyond Voxels: Tractography

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- Method for visualization/analysis
  - Integrate vector field associated with grid of principle directions
  - Requires
    - Seed point(s)
    - Stopping criteria
      - FA too low
      - Directions not aligned (curvature too high)
      - Neighborhood coherence
      - Leave region of interest/volume
  - Many methods have been published during the past decade (Basser, Mori, Westin, Vermuri, Kindlmann, Lenglet, etc.)
-





# White Matter Fiber Tract Atlases

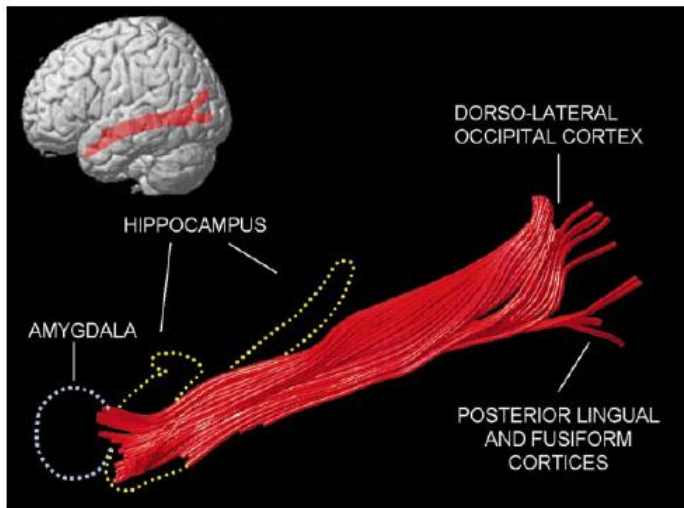


Fig. 7 Reconstruction of the ILF in the average DT-MRI data set. The long fibres originate from extrastriate areas of the occipital lobe and terminate in lateral temporal cortex and medial temporal cortex in the region of the amygdala and parahippocampal gyrus.

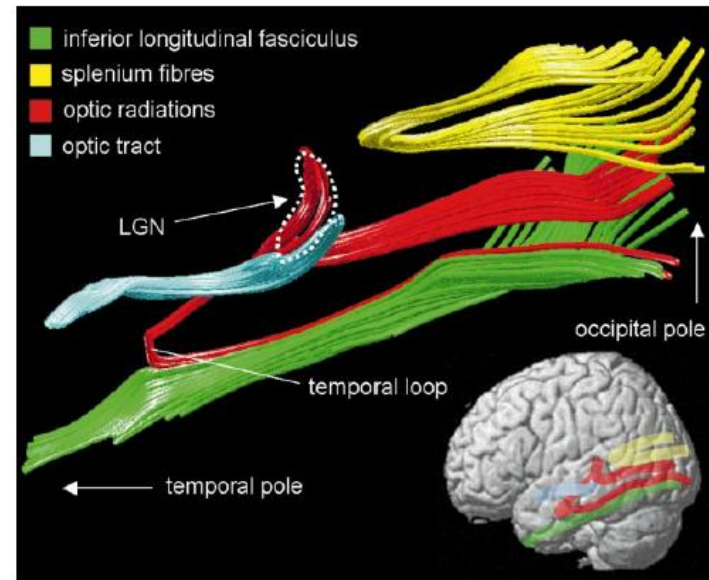


Fig. 2 Virtual *in vivo* dissection of the ILF and visual pathway of the right hemisphere (medial view) in the average brain data set. Splenial fibres connecting medial occipital regions are also shown. See text for explanation.

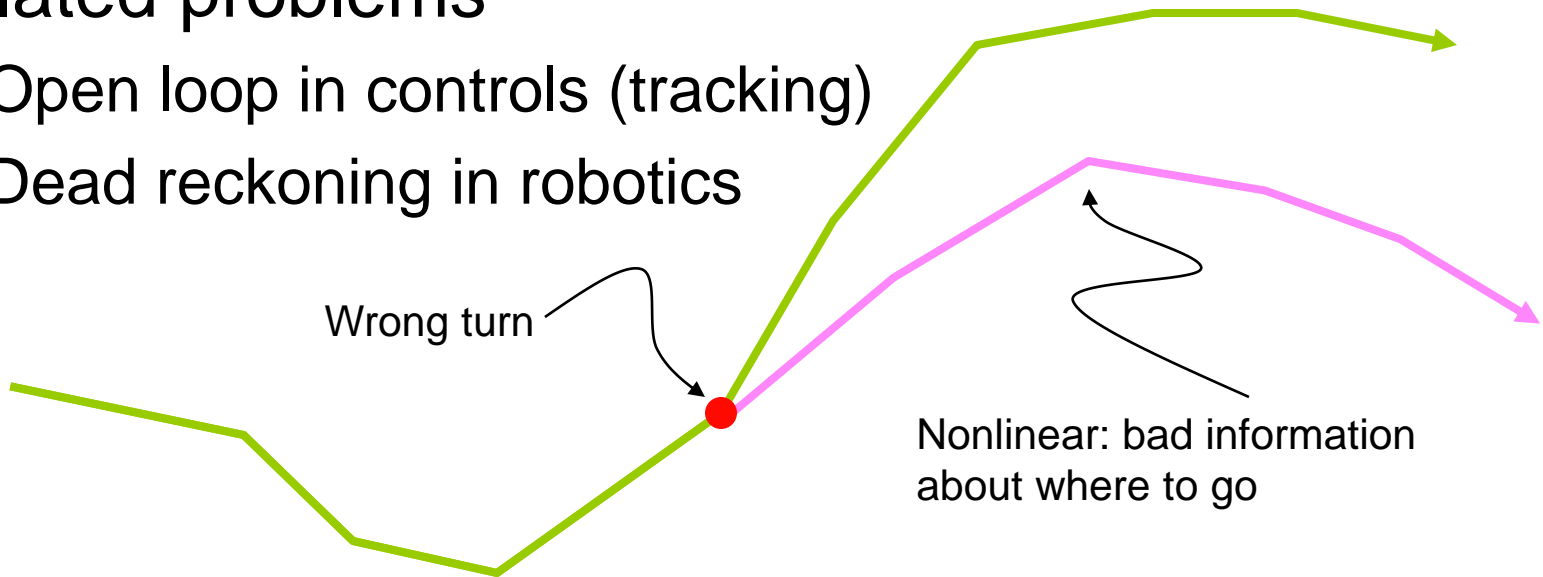
Catani et al., Occipito-temporal connections in the human brain, Brain 2003



# The Problem with Tractography

## How Can It Work?

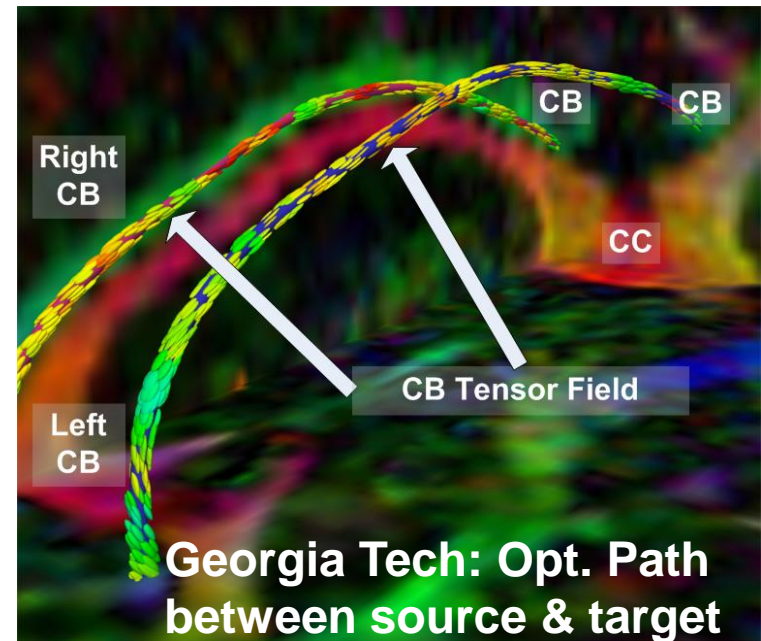
- Integrals of uncertain quantities are prone to error
  - Problem can be aggravated by nonlinearities
- Related problems
  - Open loop in controls (tracking)
  - Dead reckoning in robotics





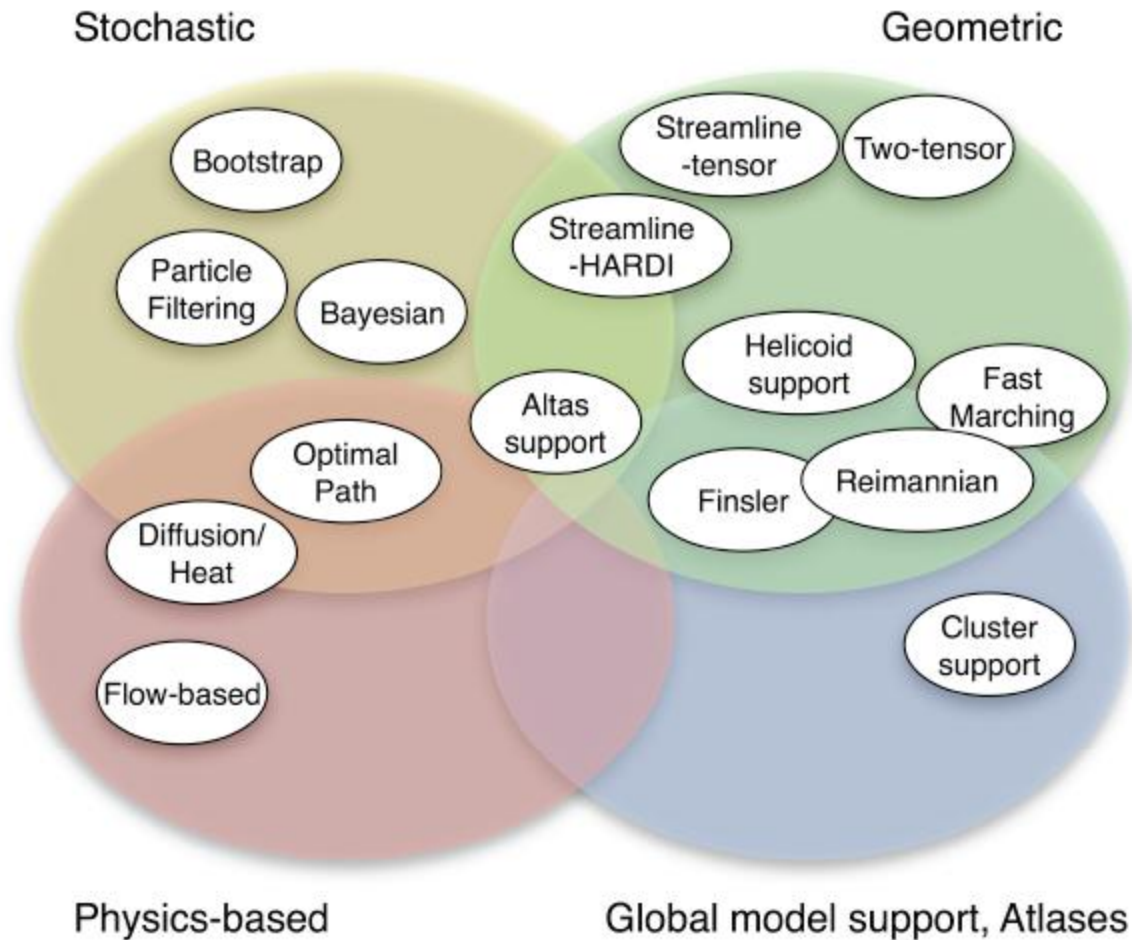
# Alternative methods for tractography

- Tracking in vector-field of largest eigenvector
- Tracking in tensor field
- Probabilistic tractography
- Optimal path analysis
- Fiber tract by volumetric diffusion
- ....
- Variety of methods developed by NAMIC developers





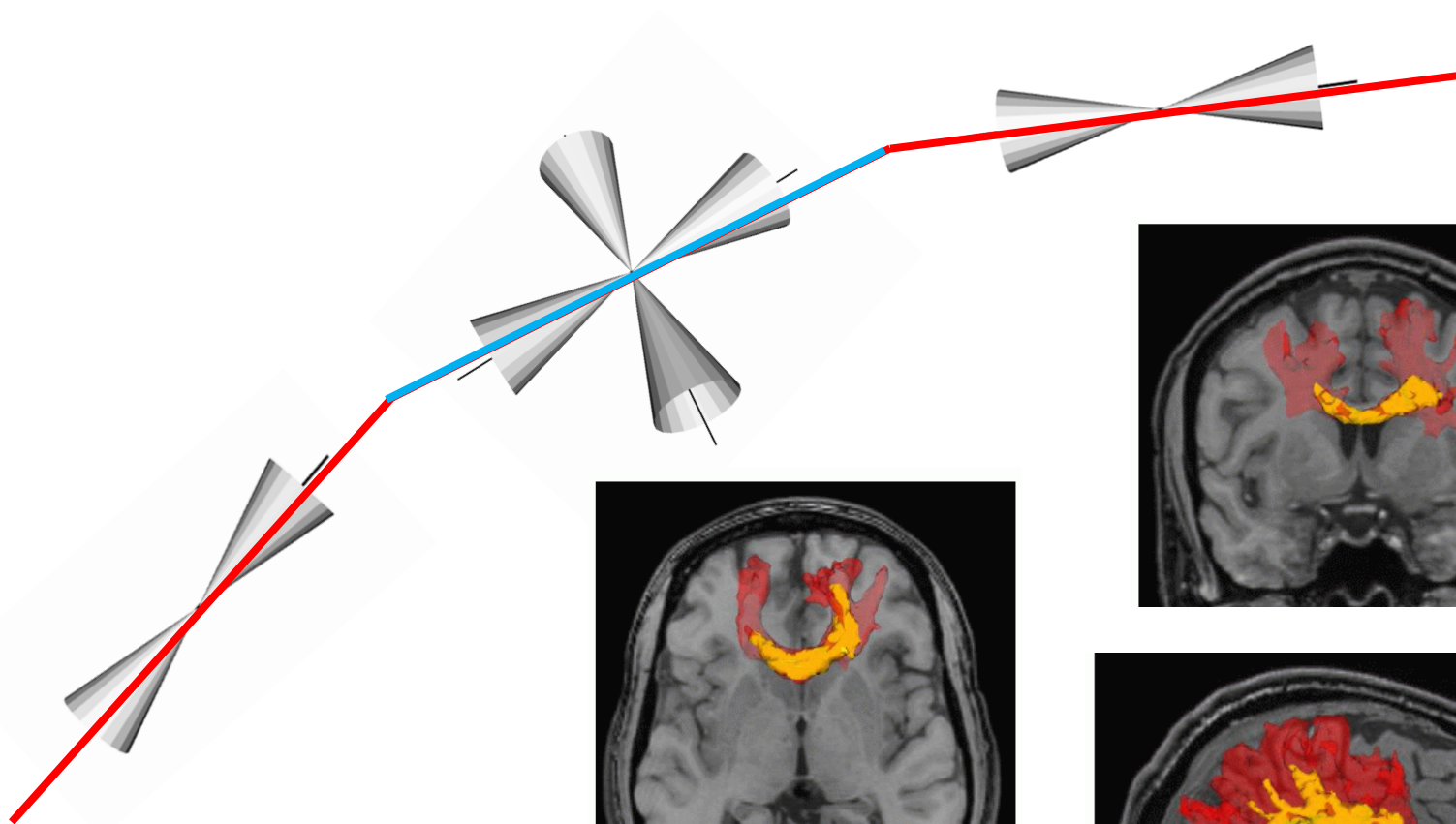
# Diffusion MRI Tractography



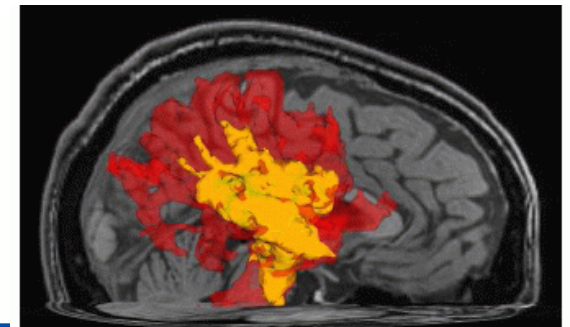
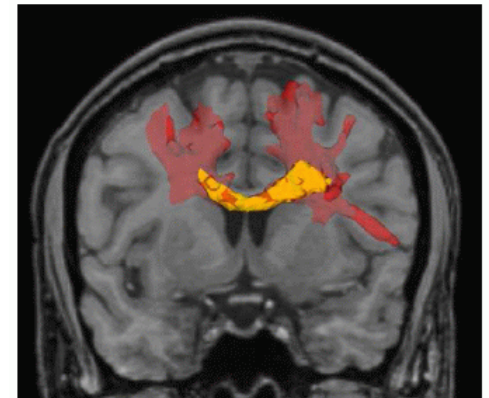
Courtesy Carl-Fredrik Westin, MICCAI 2008 workshop



# Tractography Incorporating Uncertainty



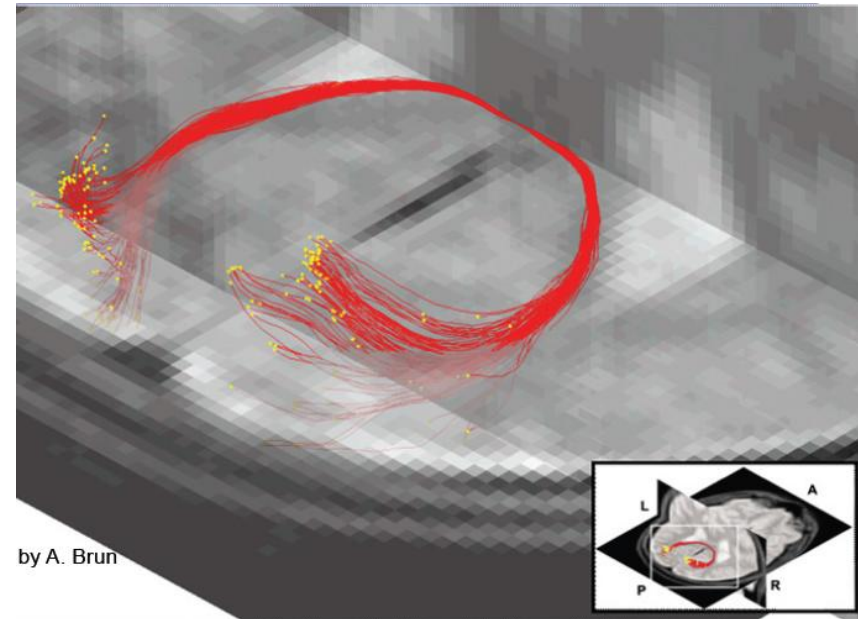
Courtesy of Bruce Pike, MNI





# Stochastic Tractography

- Lazar, Alexander, **White Matter Tractography using Random Vector (RAVE) Perturbation**, ISMRM 2002
- D. Tuch, Diffusion MRI of complex tissue structure, Ph.D. dissertation, Harvard-MIT, 2002
- Brun, Westin, **Regularized Stochastic White Matter Tractography Using Diffusion Tensor MRI: Monte Carlo, Sequential Importance Sampling and Resampling**. MICCAI 2002.
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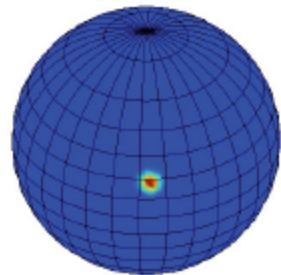
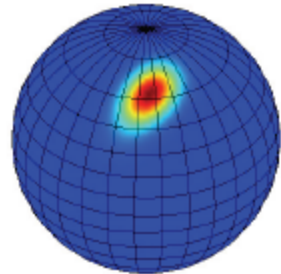
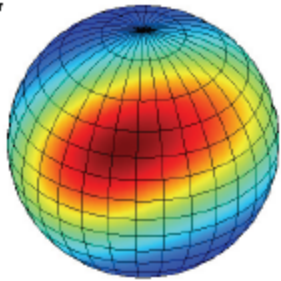
Courtesy Carl-Fredrik Westin,  
MICCAI 2008 workshop



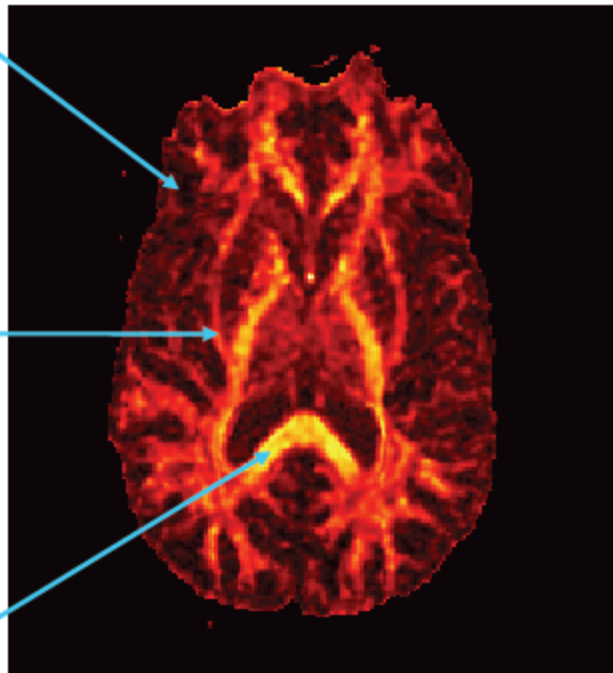
# Stochastic Tractography

laboratory of  
torner

Friman, Westin MICCAI 2005, TMI 2006



Fractional anisotropy



A probability density function of the fiber orientation in each point.

Start point



In every step, draw a step direction from the pdf of the underlying fiber orientation.

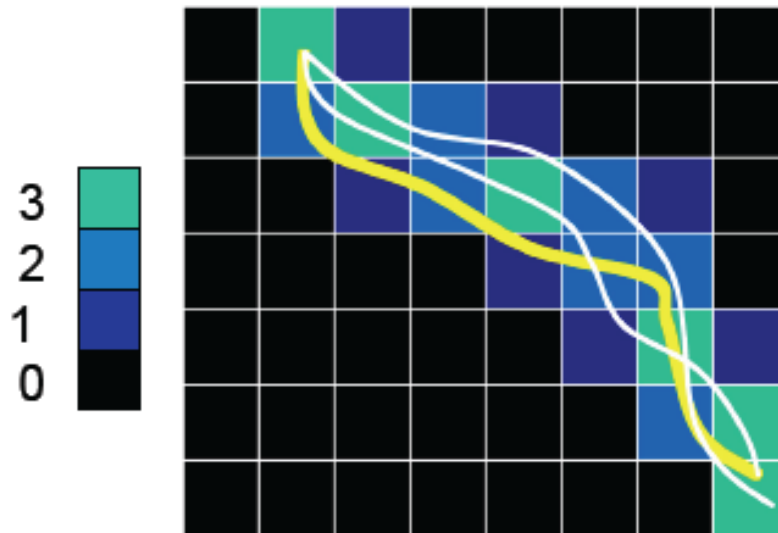
Rickman and Westin, Hospital, Harvard Medical School

Courtesy Carl-Fredrik Westin, MICCAI 2008 workshop



# Probability of Connection

Given a large number of fibers, the probability of a connection between two voxels can be estimated



Probability density function: 1) Add the contribution from all paths, and 2) normalize the total sum of all voxels

Courtesy Carl-Fredrik Westin, MICCAI 2008 workshop





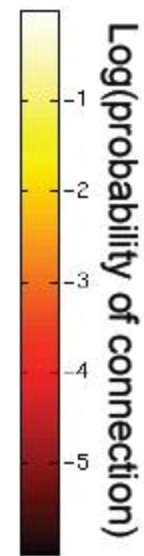
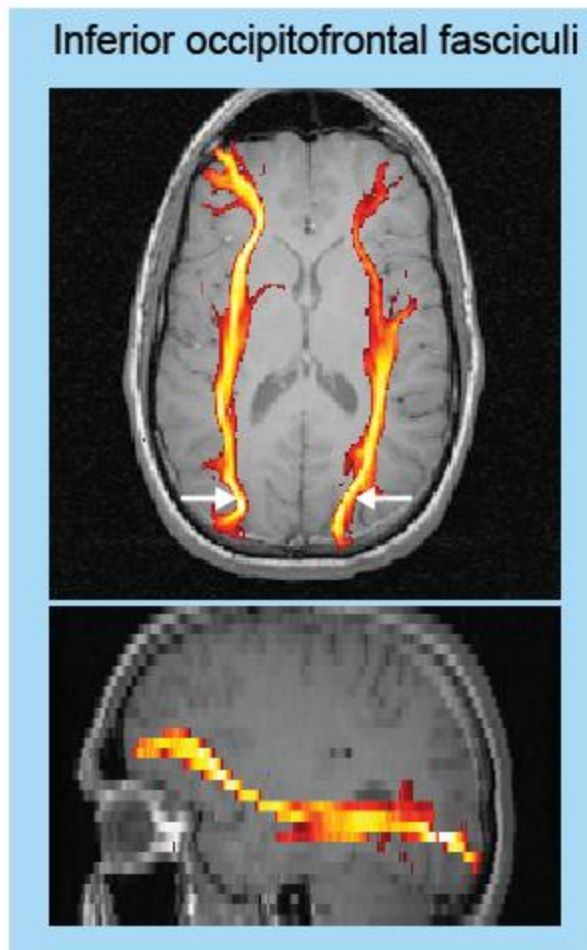
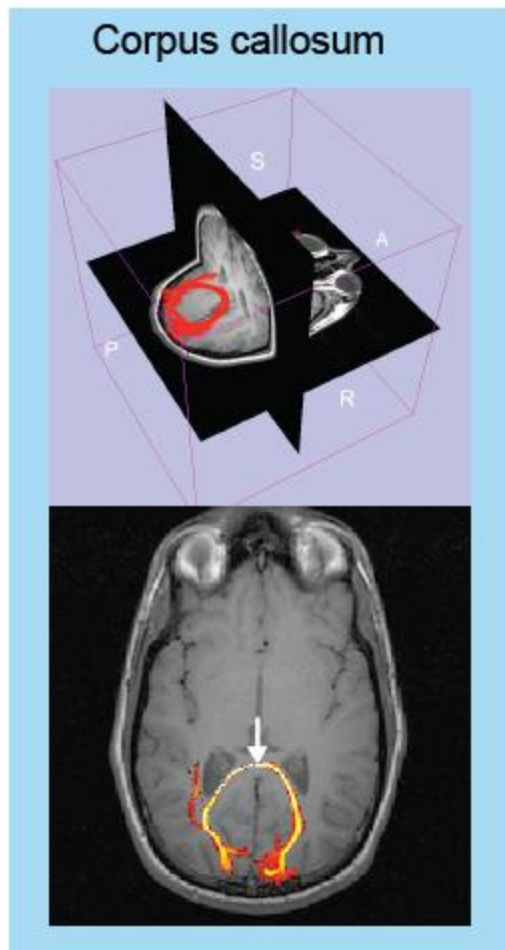
# Tractography



J. Fallon



# Probability of Connection

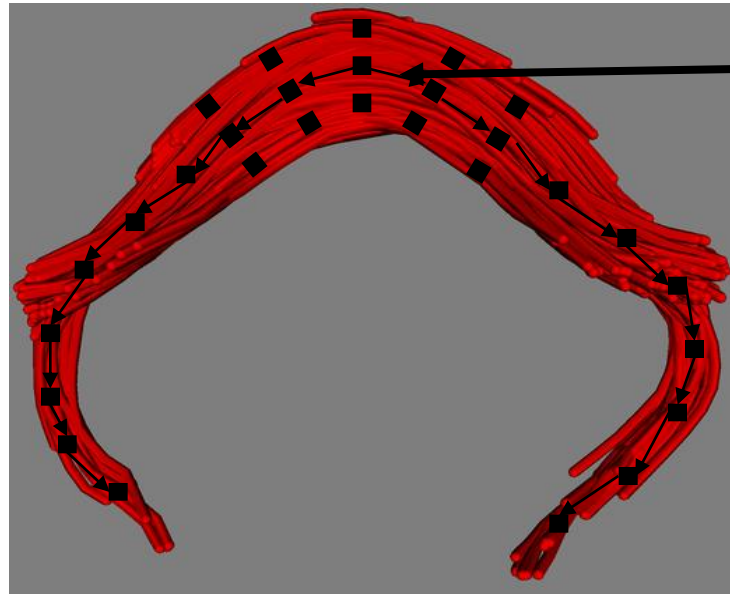


Courtesy Carl-Fredrik Westin, MICCAI 2008 workshop

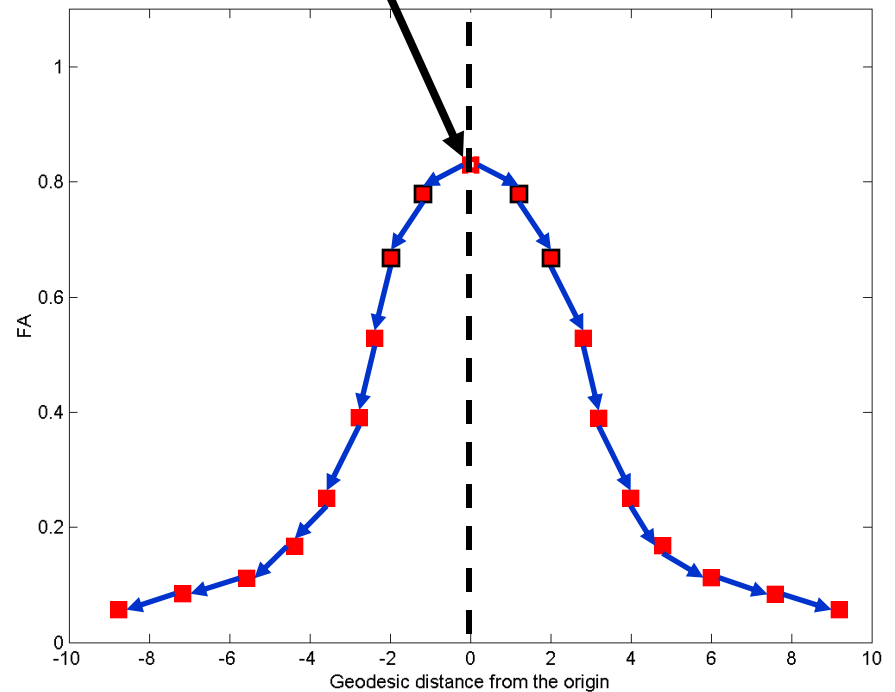
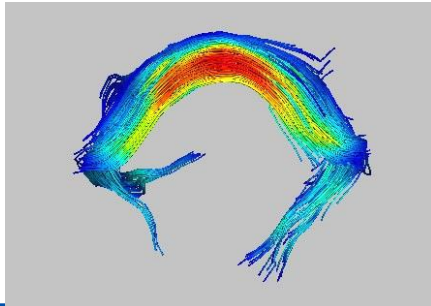
Work with O. Friman



# Modeling of fiber tracts

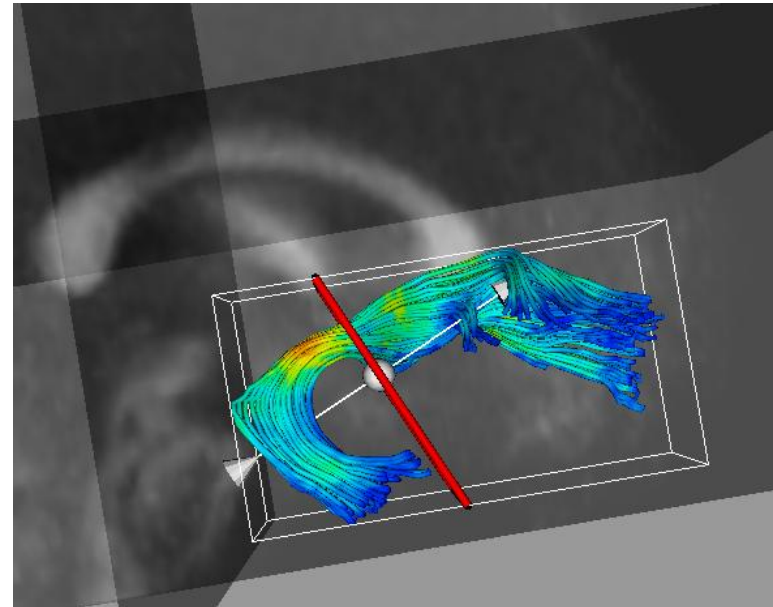
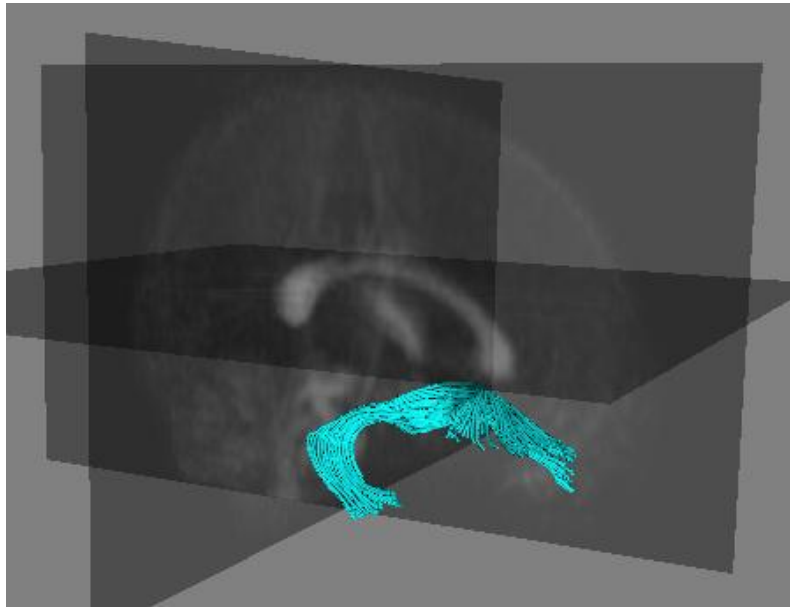


Origin (anatomical landmark)

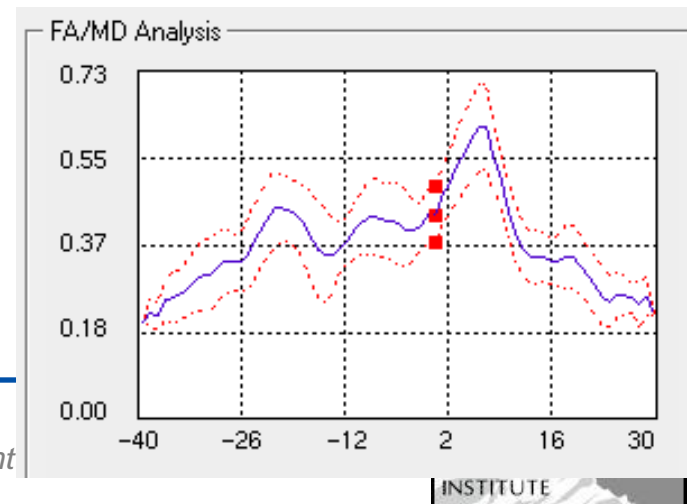




# Example Uncinate Fasciculus

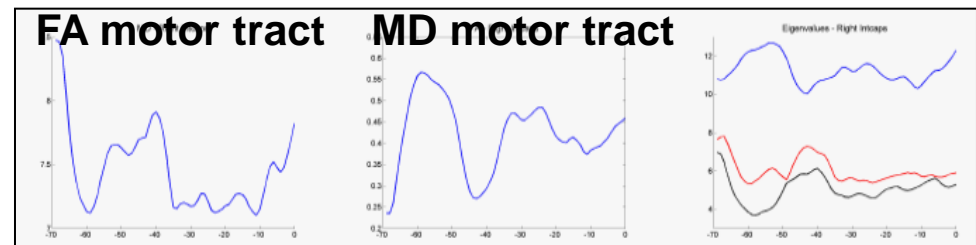
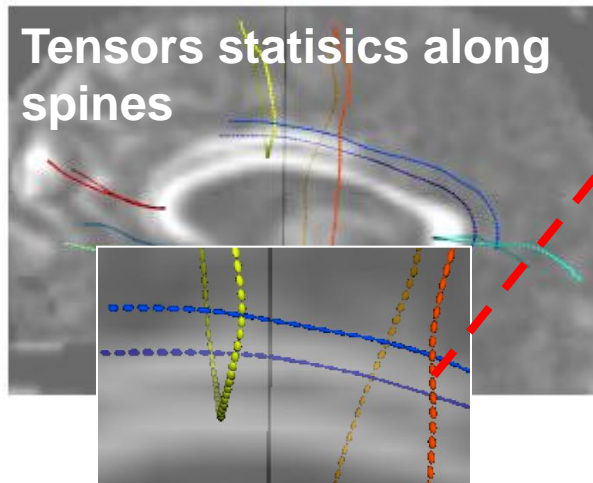
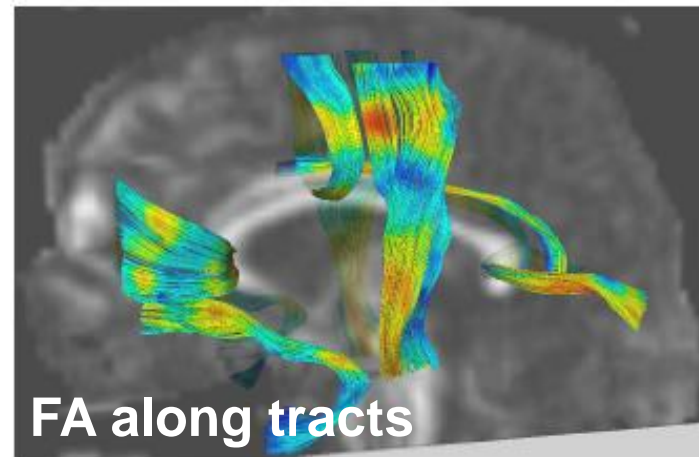
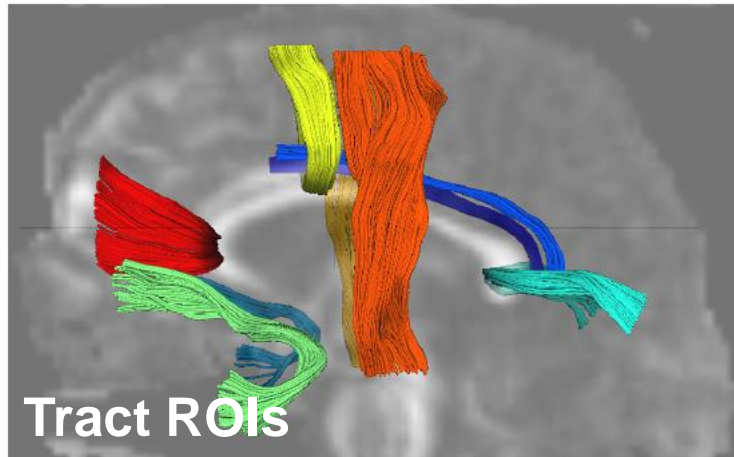


Corouge et al. *Fiber tract-oriented statistics for quantitative diffusion tensor MRI analysis*. Medical Image Analysis 2006.  
FiberViewer software - <http://www.ia.unc.edu/dev/>





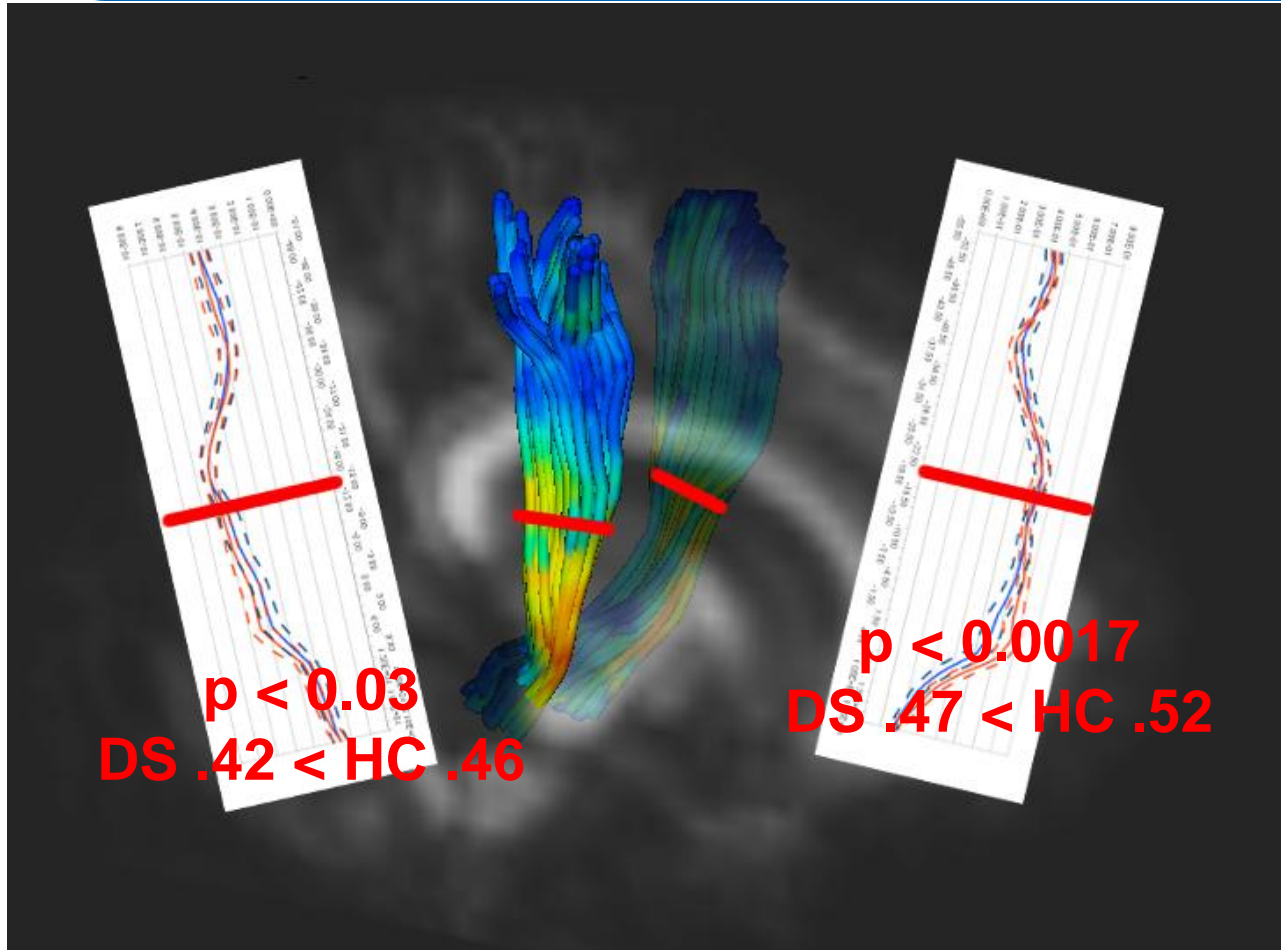
# Quantitative Tractography



- Tractography for ROI definition
- Tensor-math. for statistics along tracts



# Group testing of FA along motor tracts (Downs Syndrome vs. HC)

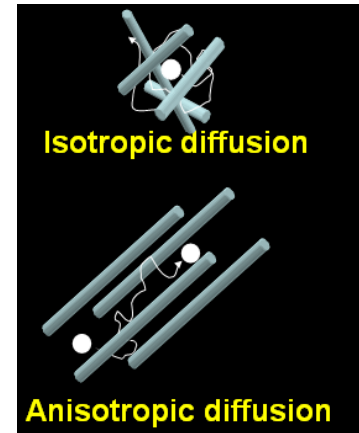


**Center:** Left and right motor tracts, color-coded with FA (0 to 1 in blue to red)

**Left and right:** FA as function of arc-length for HC (red) and DS (blue)



# Summary: What do we measure?



- DWI measures local diffusivity pattern.
- Local diffusivity pattern is shaped by tissue type, axon structuring, myelination etc.
- Curves and streamlines from tractography are NOT AXONS but possible paths in vector/tensor field.
- “Fiber counting” scientifically questionable, # is method specific.
- **DWI DOESN'T MEASURE AXONS or GLOBAL CONNECTIVITY !**



# Limitation of Tractography: Infer global structures from local estimates



We measure diffusion structure of local elements ( $2 \times 2 \times 2 \text{ mm}^3$ ) and make inference/guess about road network  $\rightarrow$  axonal bundles.

- Voxel size: cubic mm
- Axon diameter: micrometers



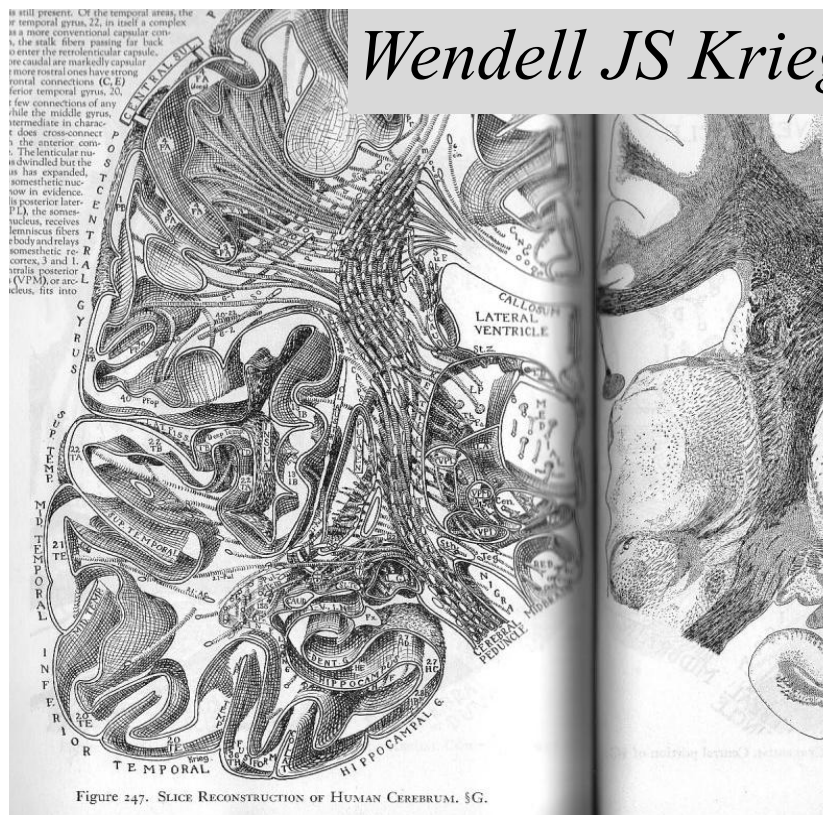
Courtesy Carl-Fredrik Westin, MICCAI 2008 workshop



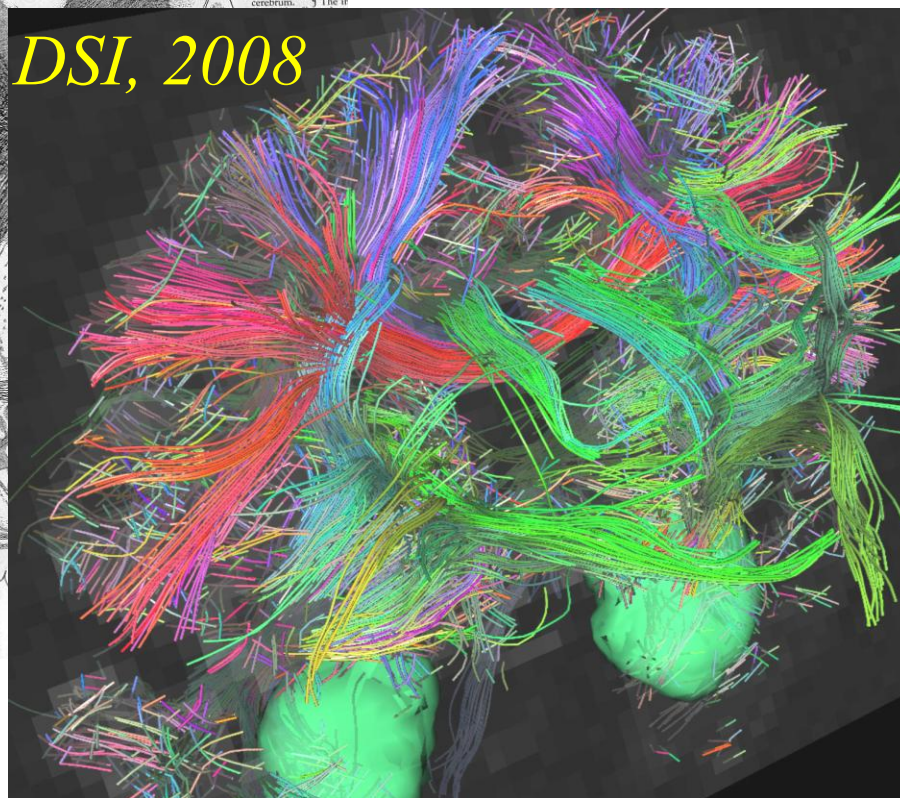


# Emerging New Techniques: HARDI, DSI, Q-Ball

*Wendell JS Krieg, 1963*



*DSI, 2008*



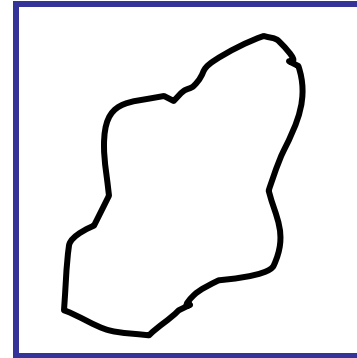
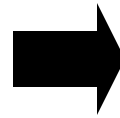
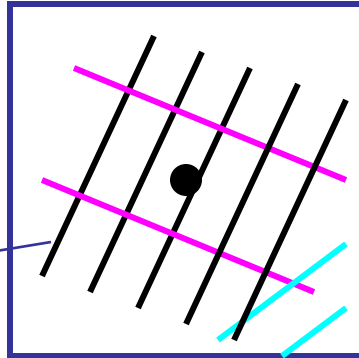
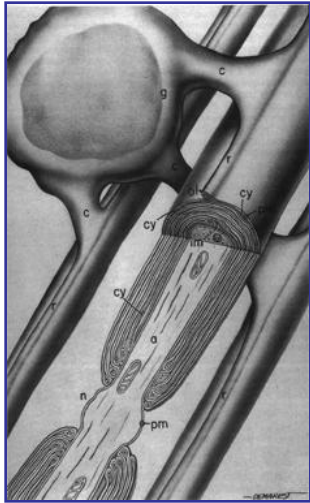
is still present. Of the temporal areas, the middle temporal gyrus, 22, is itself a complex, as a more conventional capsular gyrus, the stalk fibers passing far back to enter the retroolfactory capsule. Its caudal end is markedly capular, and more rostral areas have strong frontal connections (C, E) to the anterior temporal gyrus, 23. A few connections of any kind rule the middle gyrus, intermediate in character to the anterior and posterior connections. The lentacular nucleus is divided but the middle has expanded, somewhat, as now in evidence. In posterior lateral (P.L.), the somatosensory nucleus, receives geniculate fibers from the body and relays them to the somatosensory cortex, 3 and 4. A small nucleus posterior (VPM) or nucleus, fits into the middle of the lateral ventricle.

Figure 247. SLICE RECONSTRUCTION OF HUMAN CEREBRUM. \$G.

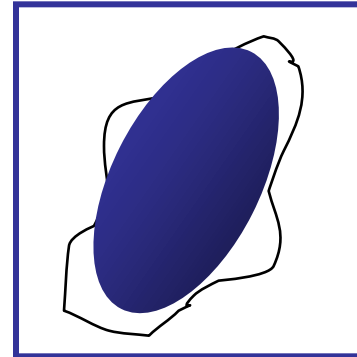
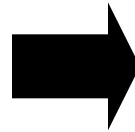
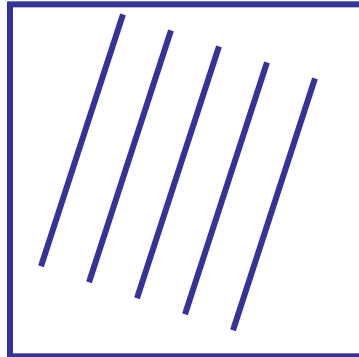
VJ Wedeen, R Wang, T Benner  
MGH-Martinos Center, Harvard Medical School



# Simplification and assumption



**Orientational Diffusion Fct**

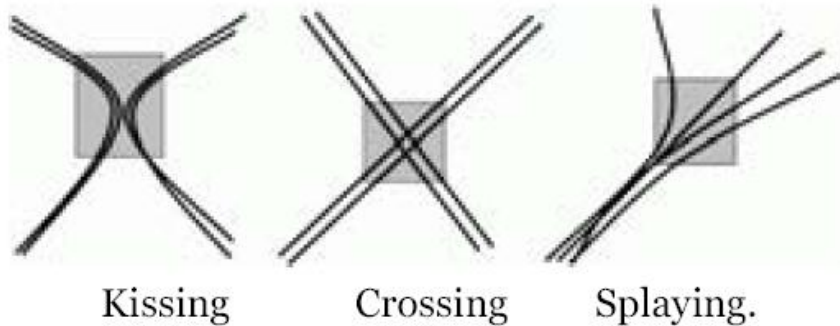


**Diffusion ellipsoid**

Courtesy of Susumu Mori, JHU



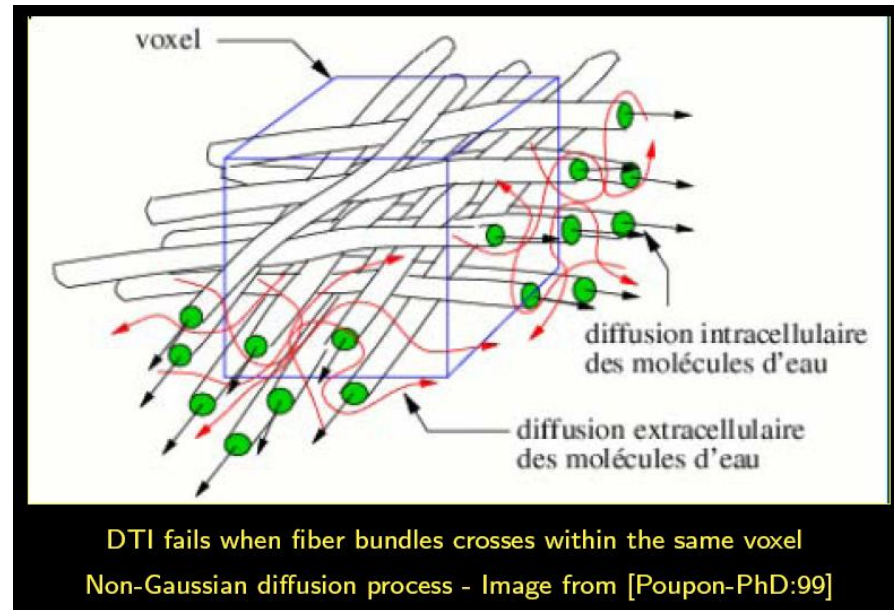
# Limitations of the Diffusion Tensor Model



Diffusivity in a fiber crossing



2nd-order tensor approximation

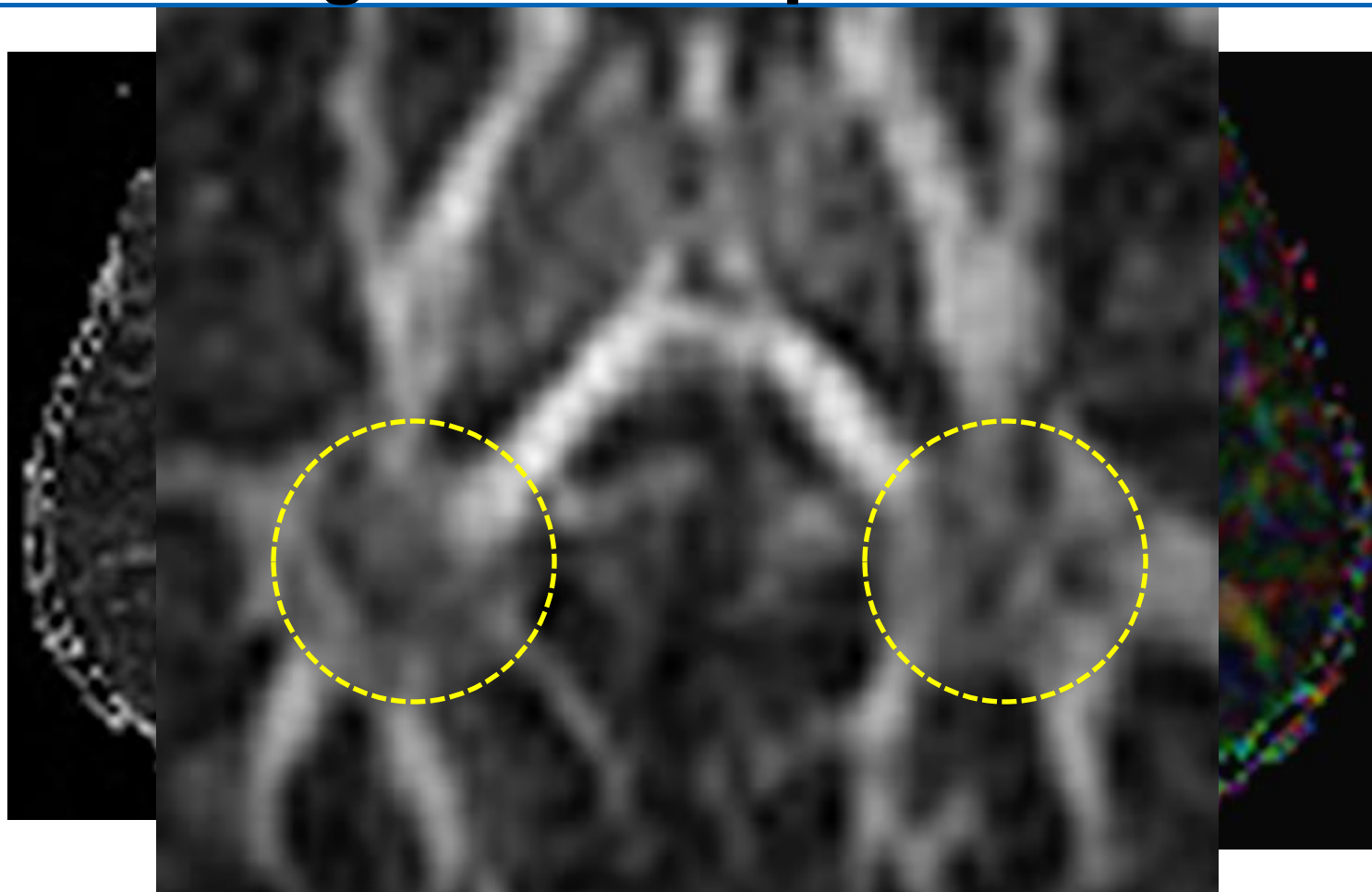


Courtesy B. Vemuri, MICCAI 2008 workshop



# DTI with 12 directions & 2 averages

## Crossing Fibers Dropout on FA



Courtesy R. McKinstry



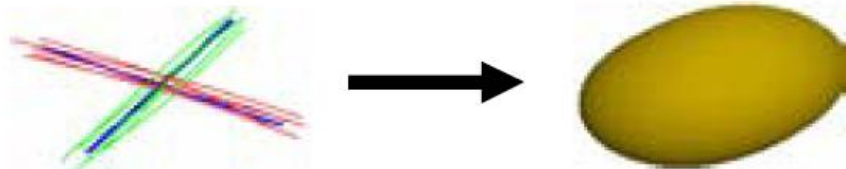
# Motivation: Why higher order?

- 2<sup>nd</sup> order tensors are popular for representing the diffusivity profile in DW-MRI data sets.

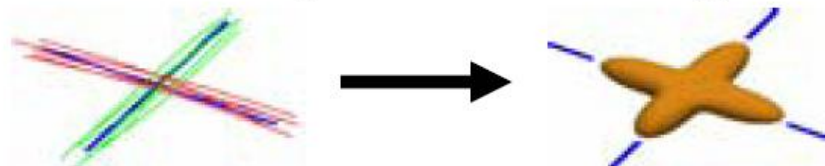
$$d(\mathbf{g}) = \mathbf{g}^T \mathbf{D} \mathbf{g}$$

$\mathbf{g} = [g_1 \ g_2 \ g_3]^T$  is the magnetic field gradient direction,  $\mathbf{D}$  is the estimated tensor.

- 2<sup>nd</sup> order tensors fail to represent fibers crossings.



- Higher order tensors can capture fiber crossing geometry.



Courtesy Baba Vemuri,  
MICCAI 2008 workshop



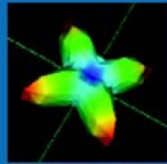
# Orientation Distribution Function ODF

**ODF and FRT allows to effectively recover the fibers direction**

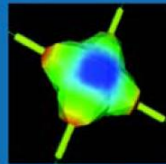
1. Apparent Diffusion Coefficient (ADC)
2. Orientation Distribution Function (ODF)



Fiber distribution

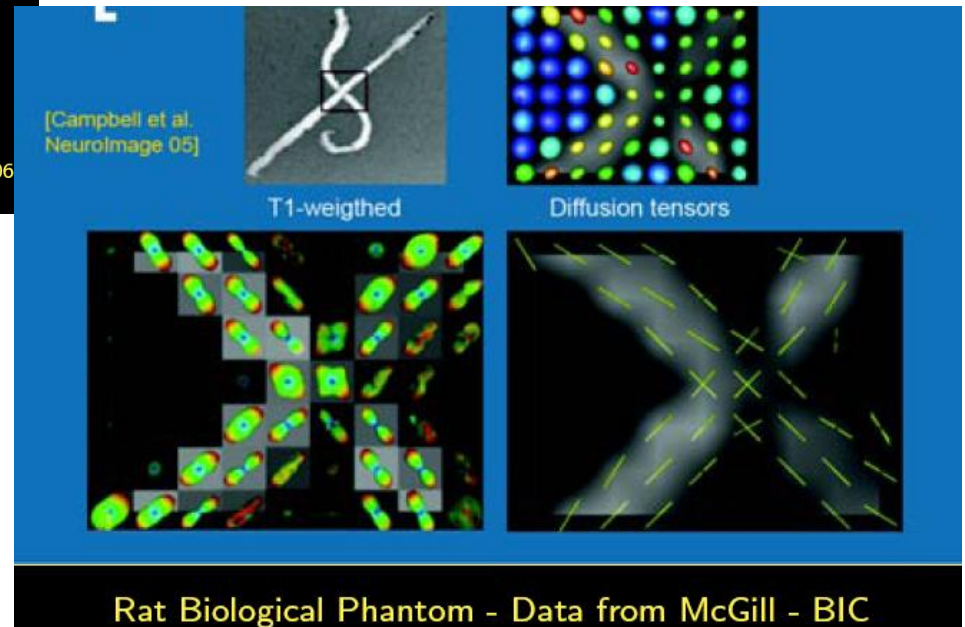


ADC profile



Diffusion ODF

Descoteaux/Angelino/Fitzgibbons/Deriché in *Magnetic Resonance in Medicine*, 2006 and 2007

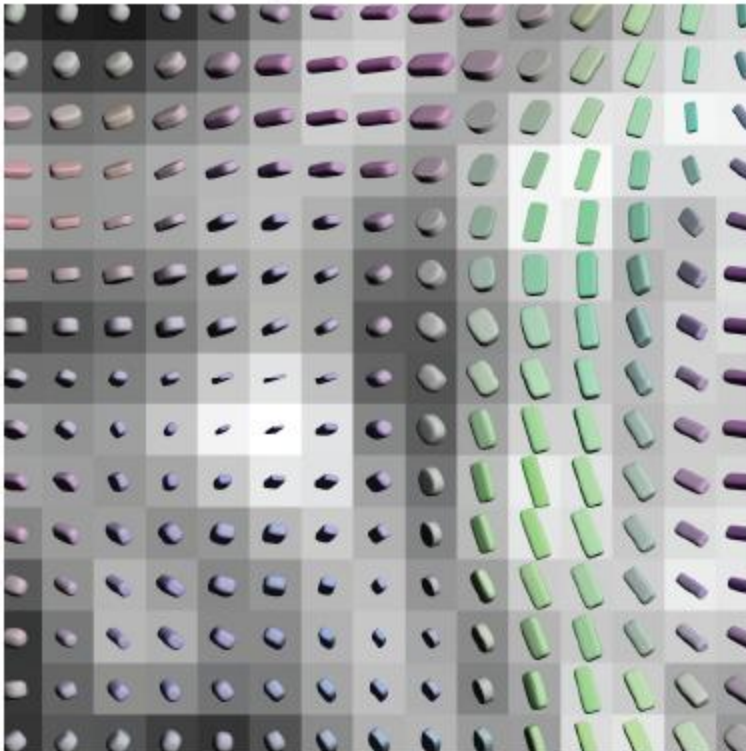


Courtesy Rachid Deriché, MICCAI 2008 workshop

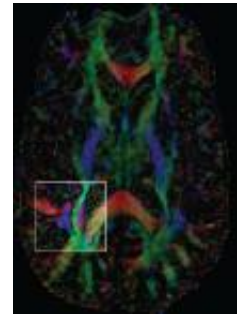
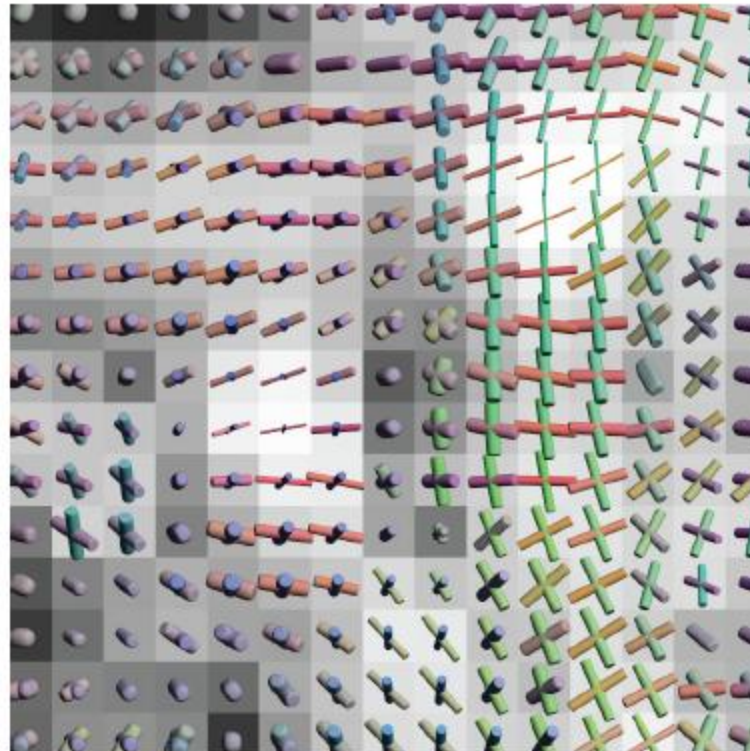


# Two Tensor Model (C-F Westin, S Peled, G Kindlmann)

DTI: Single tensor model



Two-tensor model



Courtesy Carl-Fredrik Westin, MICCAI 2008 workshop

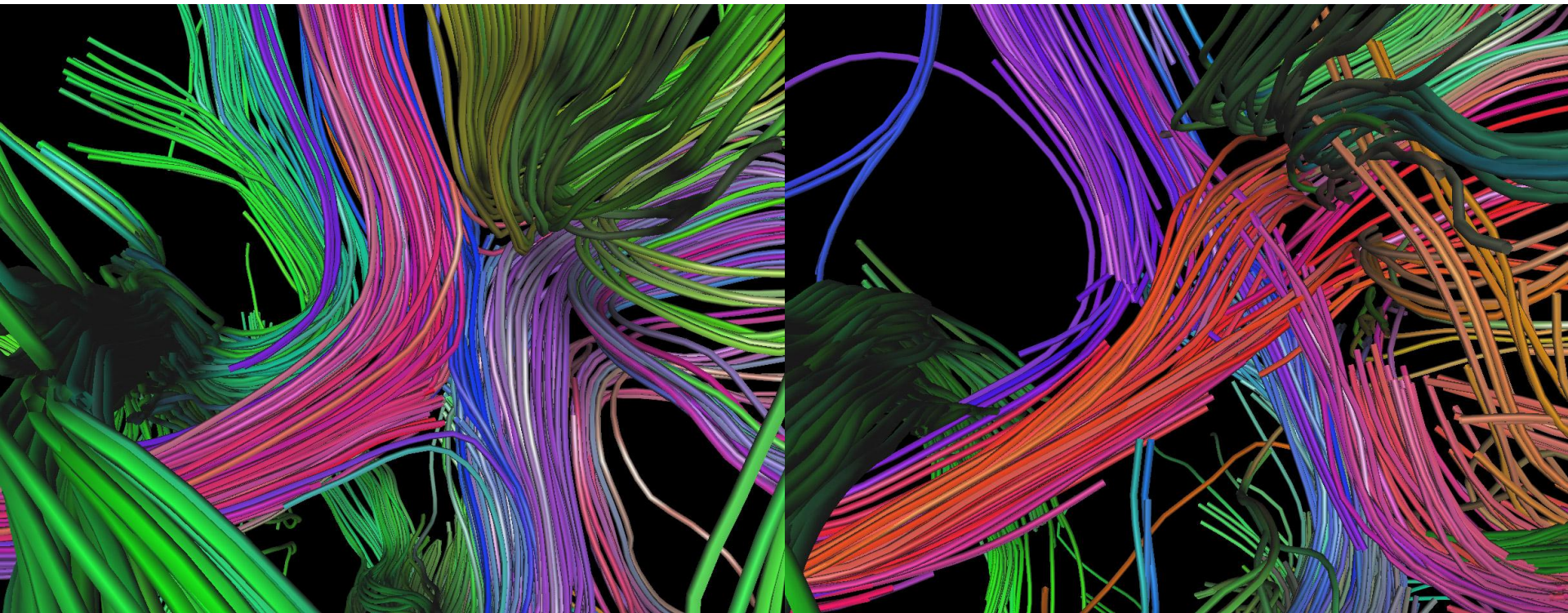


# Centrum Semiovale

corpus callosum (red) & corticospinal tract (blue)

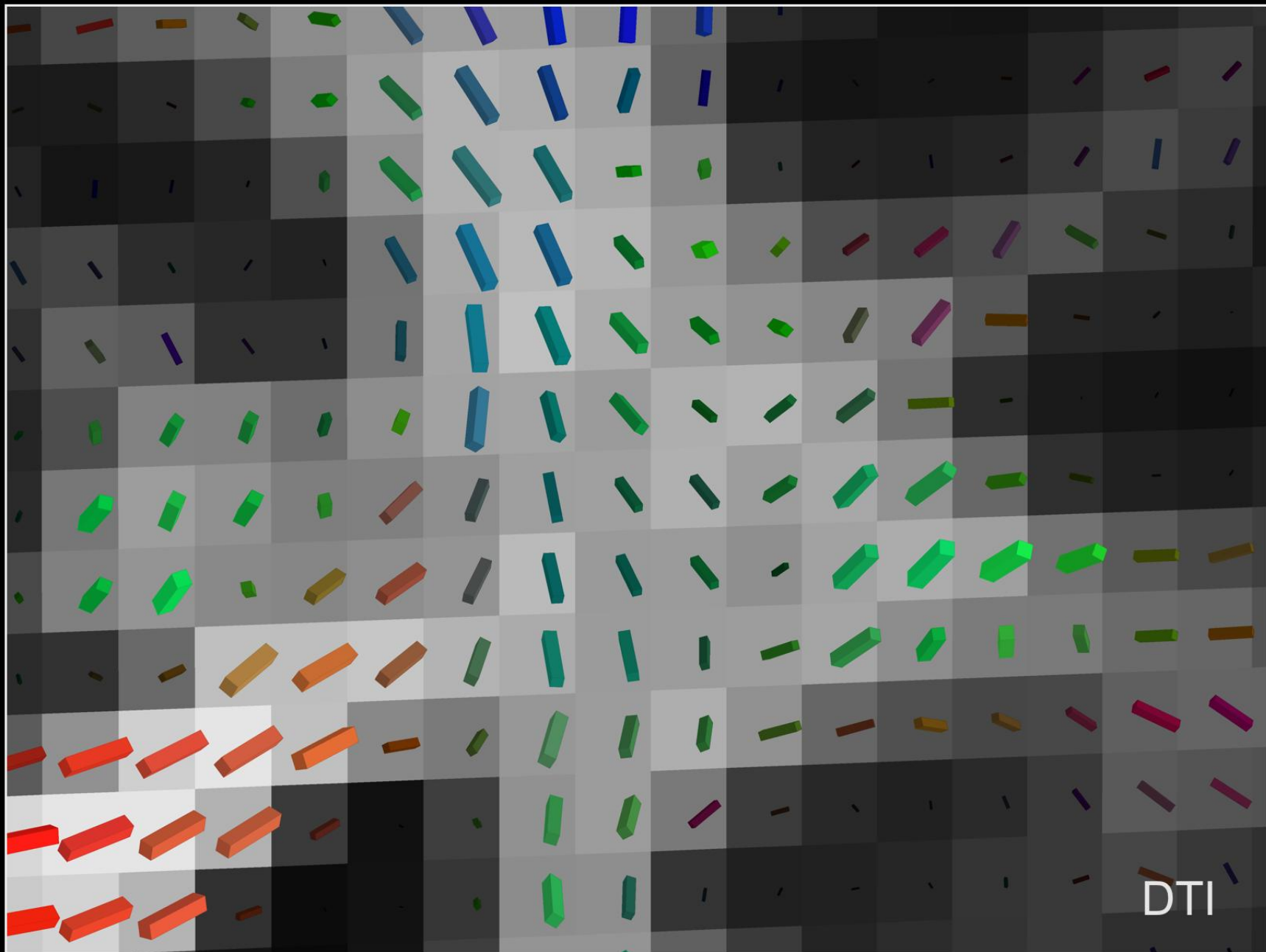
*DTI*

*DSI*



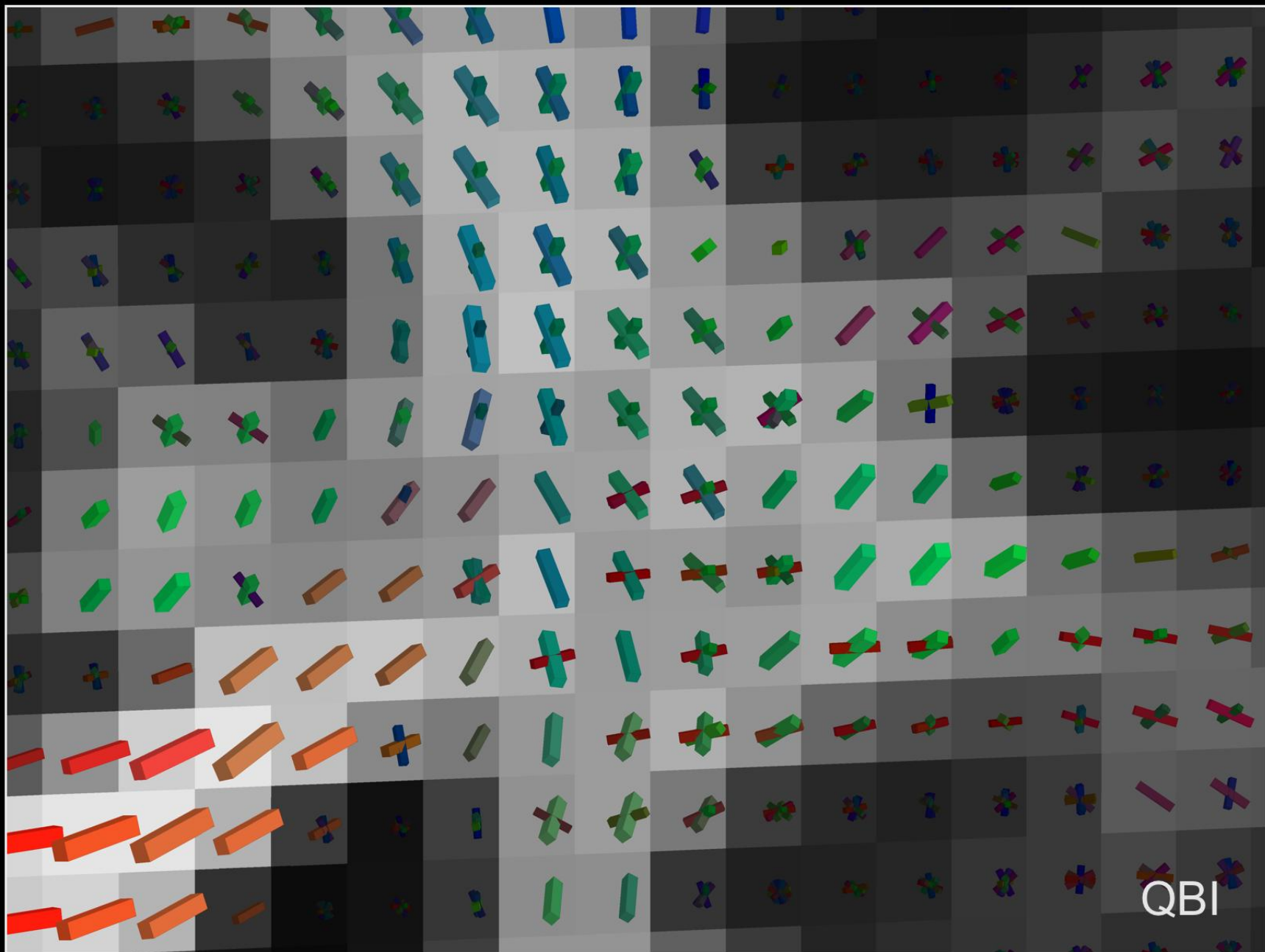
Wedeen / Wang / AGS / MGH - HST





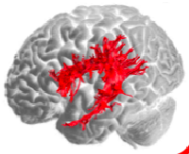
DTI

MGH / DT



MGH / DT

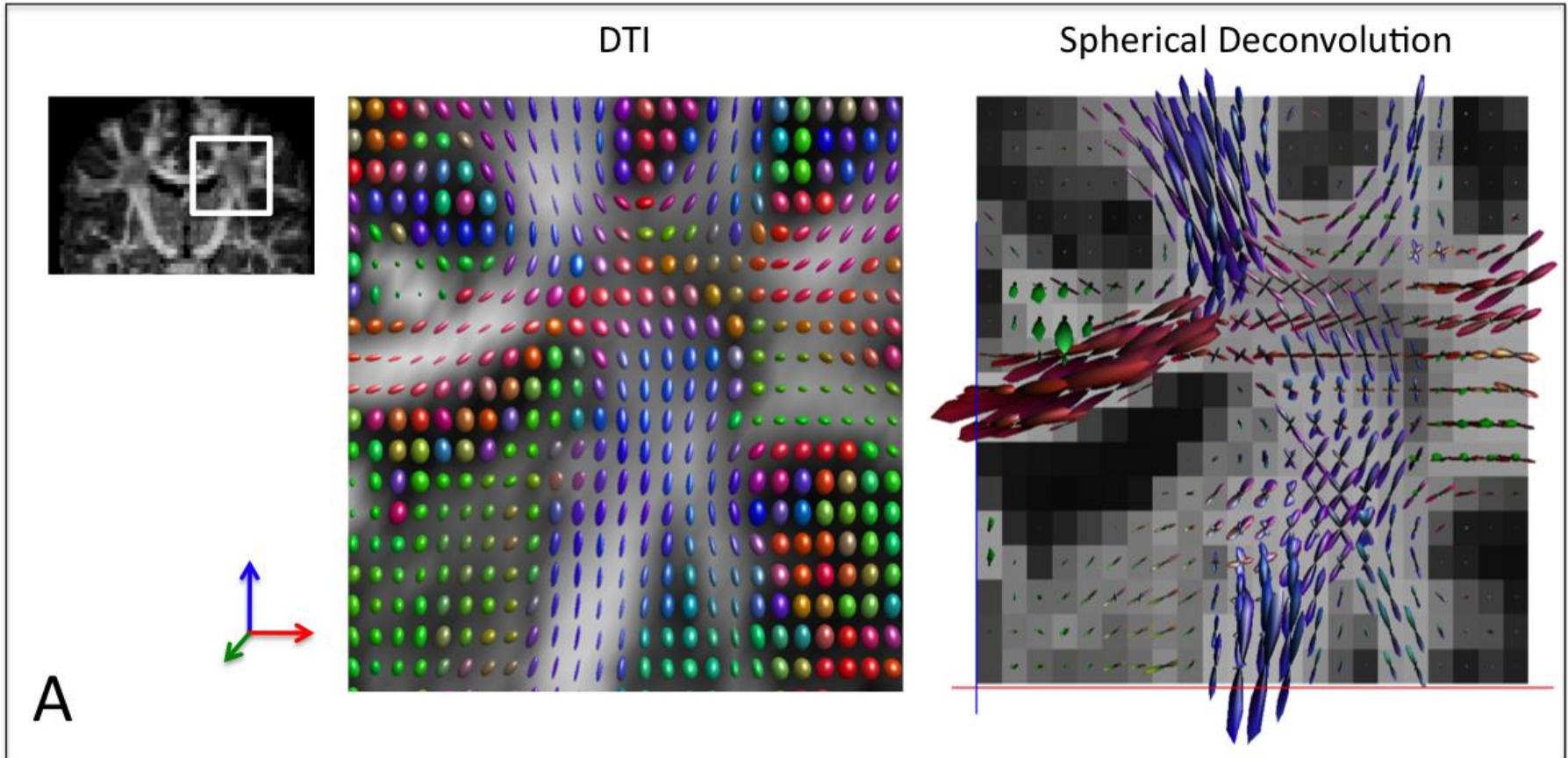
QBI



**NATBRAINLAB**

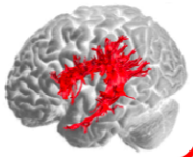
Neuroanatomy And Tractography Laboratory

# Spherical Deconvolution Tractography



60 DW directions, Cardiac Gated, Scan time = 20 min,  $b = 3000 \text{ s/mm}^2$

Dell'Acqua et al. NeuroImage 2010



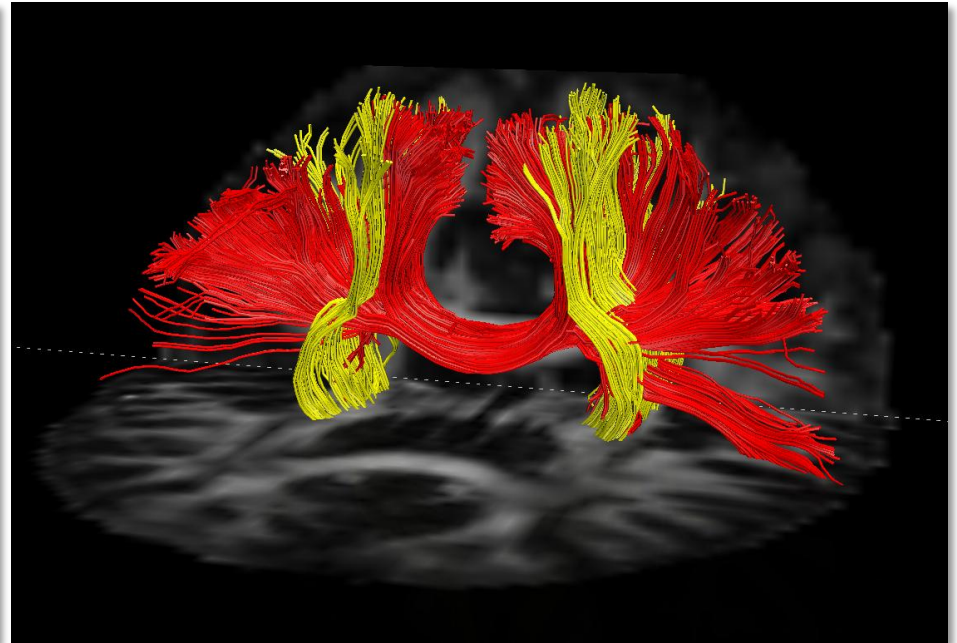
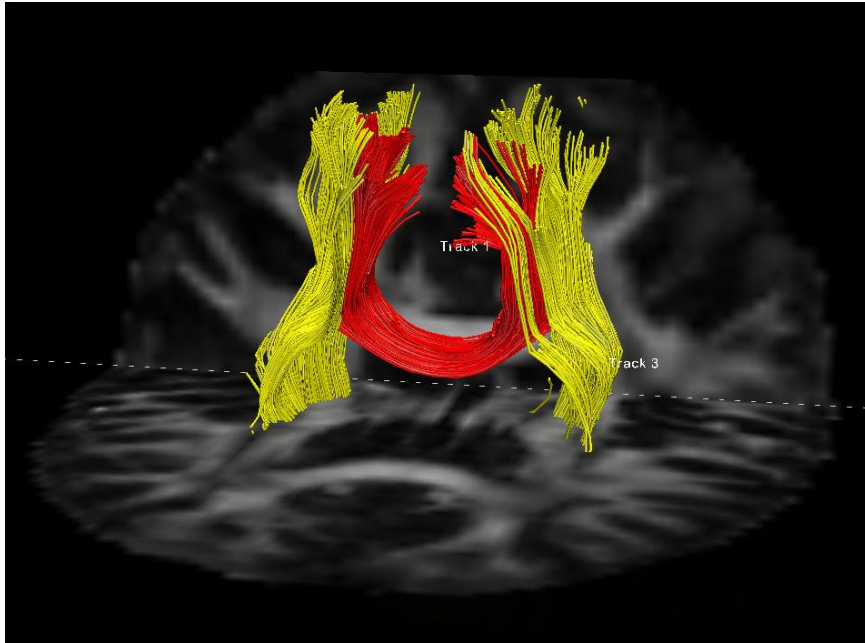
**NATBRAINLAB**

Neuroanatomy And Tractography Laboratory

*Spherical Deconvolution Tractography*

DTI Tractography

Spherical Deconvolution Tractography

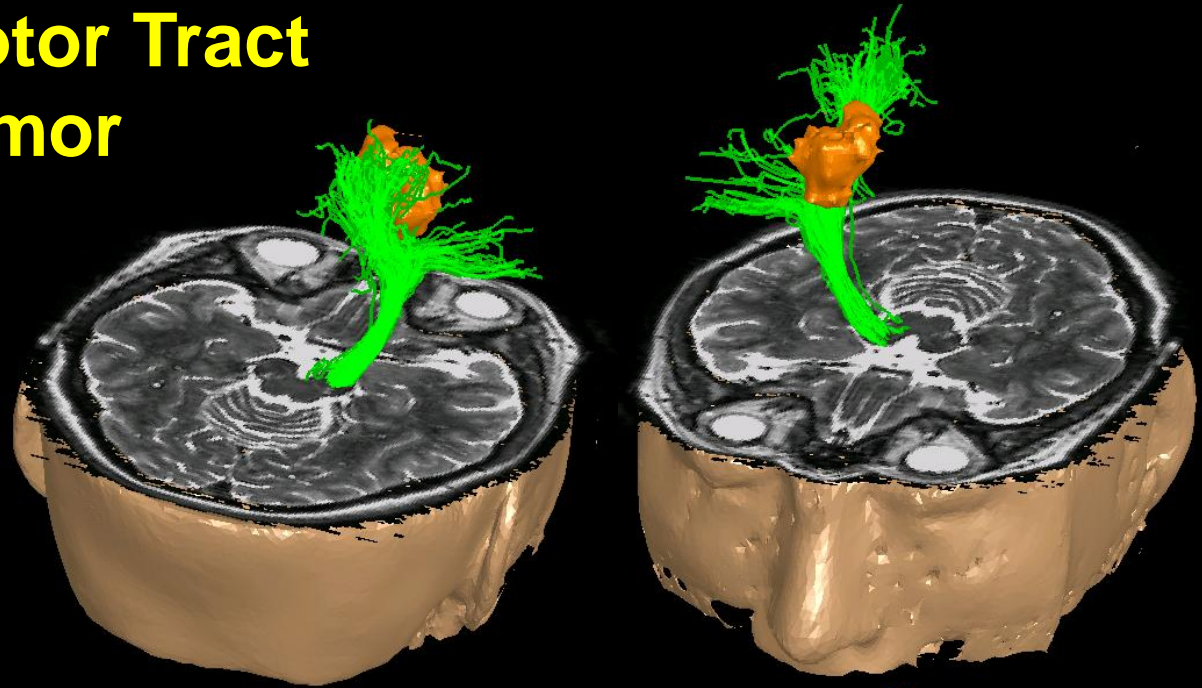


Close to standard DTI protocols →

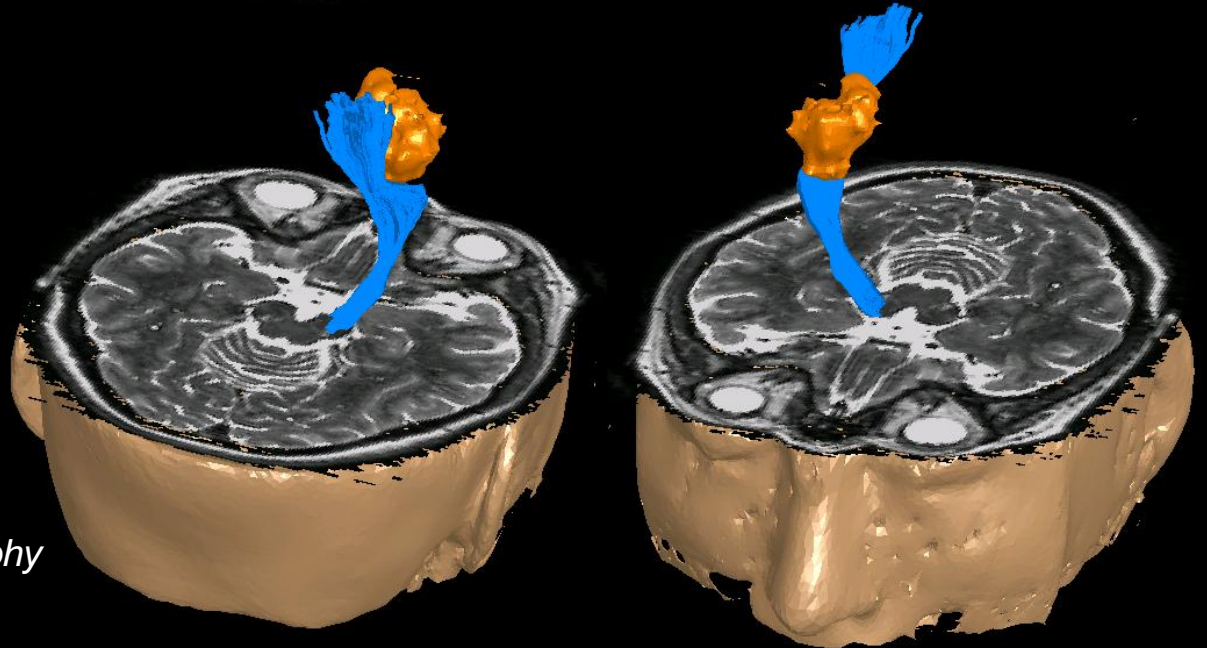
60 DW dir, Cardiac Gated, **b=1500** s/mm<sup>2</sup>

# Delineation of Motor Tract in Adult Brain Tumor Patient

Q-Ball

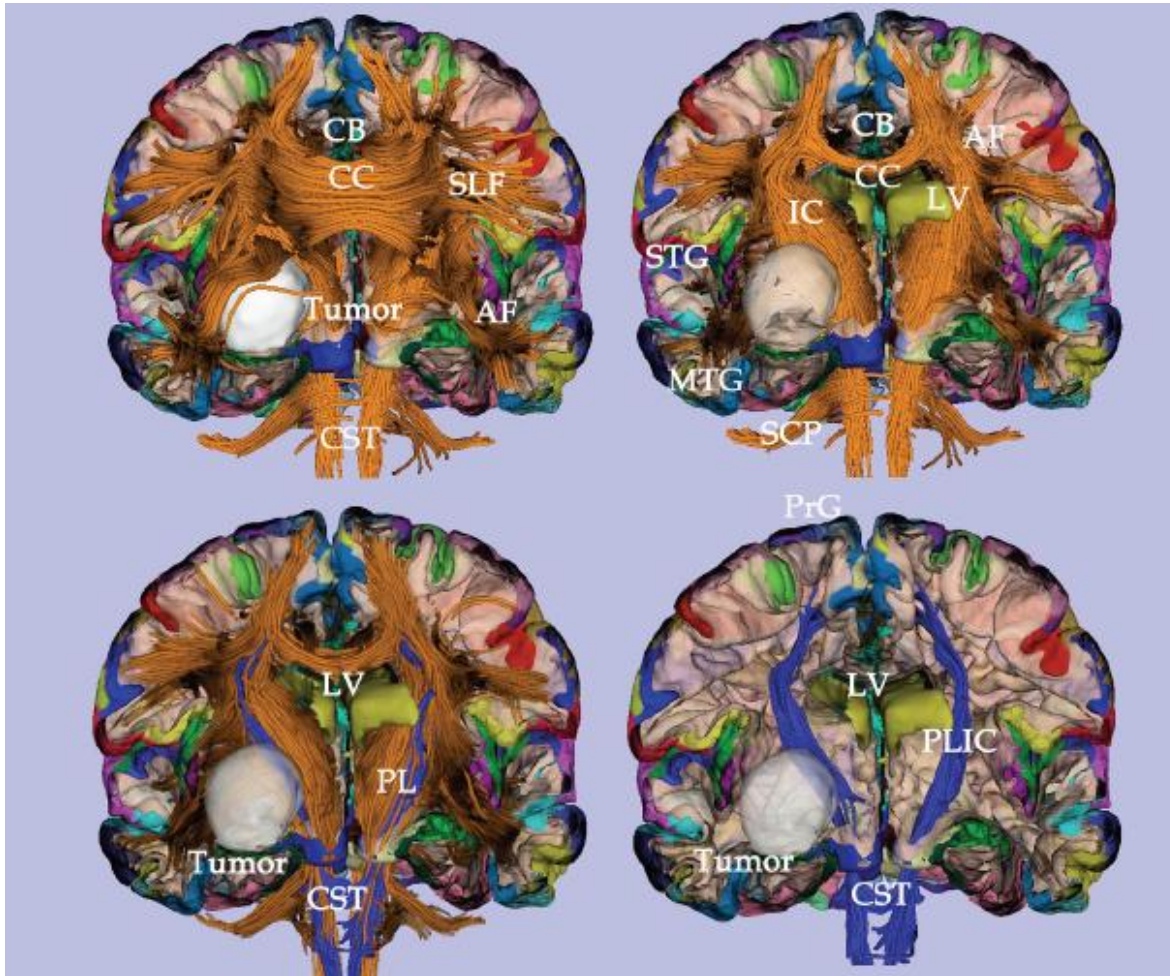


DTI





# Brain Tumor Case



Tractography based on higher-order tensor models and modeling of crossings and dispersing bundles is vital for neurosurgical applications.

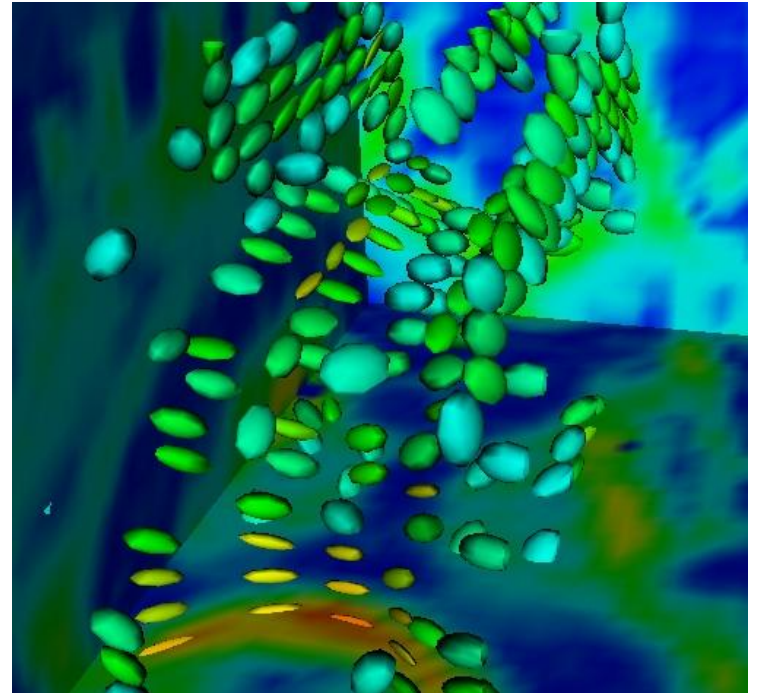
Courtesy Carl-Fredrik Westin,  
MICCAI 2008  
workshop



# Acknowledgments

## Contributors:

- C-F. Westin
- A. Alexander
- G. Kindlmann
- L. O'Donnell
- C. Goodlett
- J. Fallon
- R. Whitaker
- R. McKinstry



National Alliance for Medical Image Computing (NIH  
U54EB005149)