Recent Advances in Particle Correspondence

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http://www.na-mic.org/Wiki/index.php/Particle_UNC_Utah

Acknowledgements

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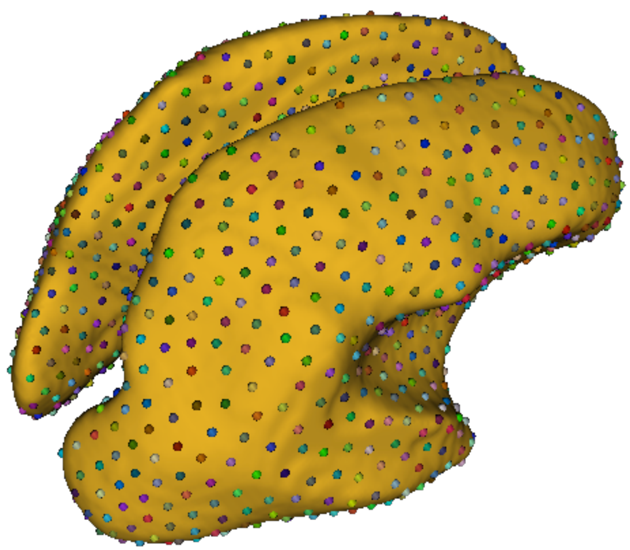
- …

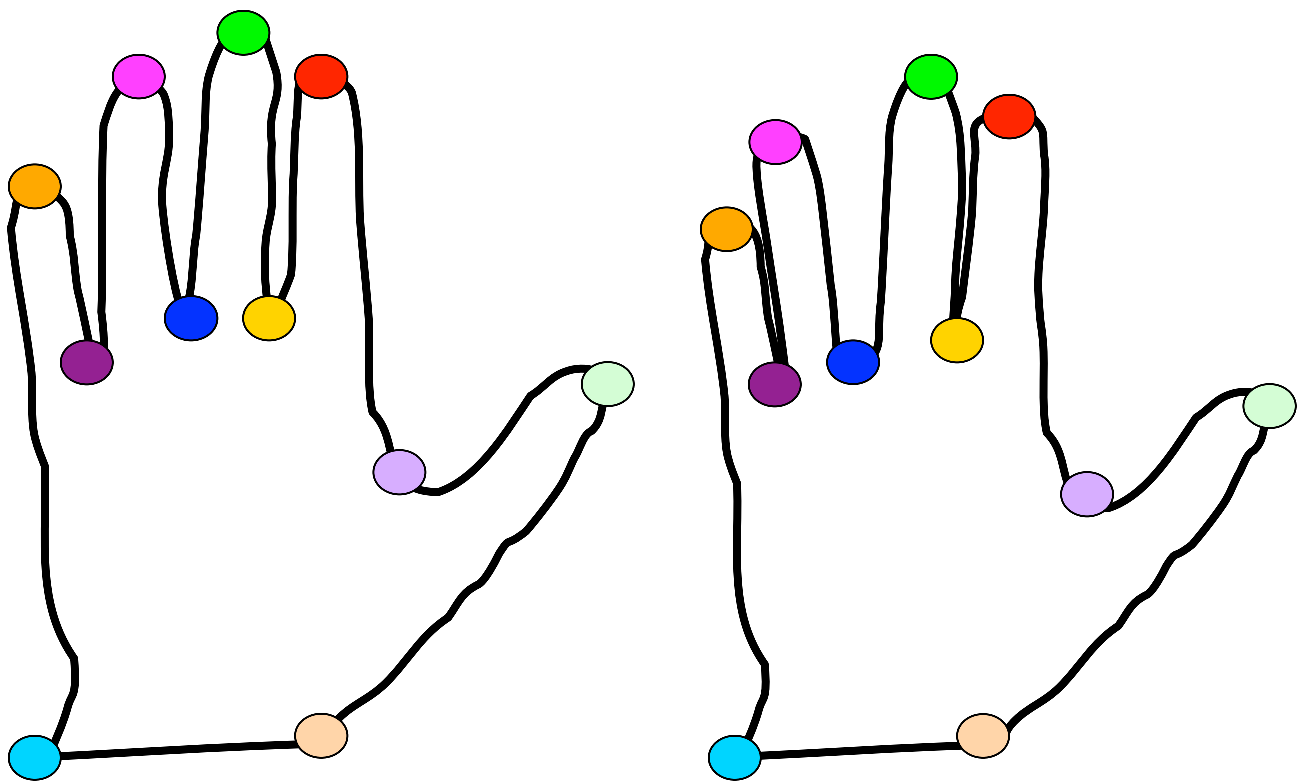
Motivation

- Statistical shape analysis
  - Structural changes in disease
  - Understanding growth process
- Correspondence key to statistical shape analysis



Correspondence Problem

- How to choose points on a surface such that they correspond across a population of surfaces?



Plan for today

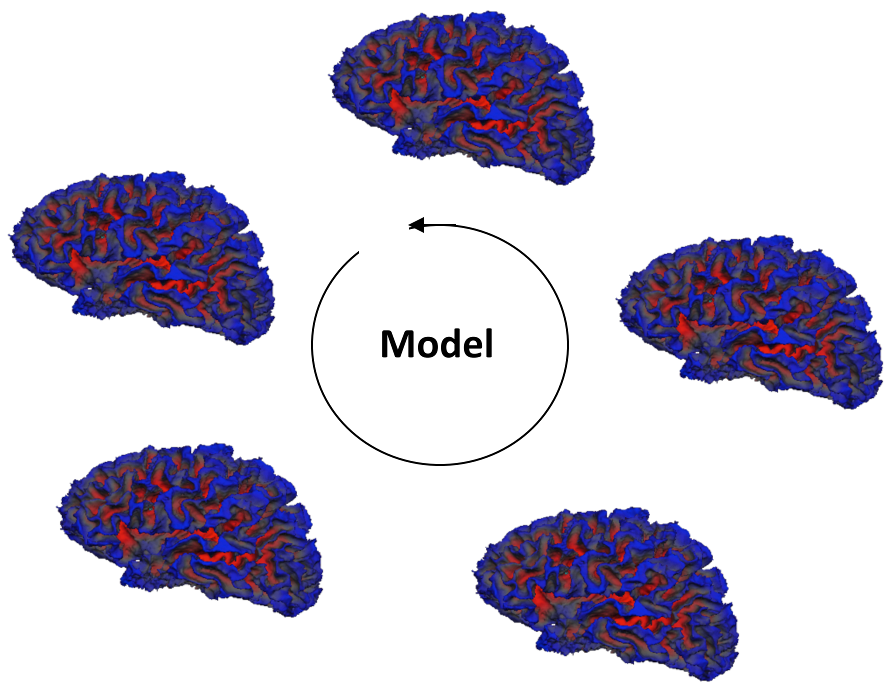
- Review of the basic idea
- Extensions
  - Generalized correspondence
  - Adaptive sampling
  - Geodesic distances
  - Normal consistency
  - Open surfaces
  - Multi-object complexes
  - Regression
- Initialization methods
- ShapeWorks
- Applications

Correspondence Overview

- Parametric vs nonparametric
  - MDL vs entropy
- Pair-wise vs group-wise
  - FreeSurfer vs MDL, entropy
- Surface-based vs volume-based
  - Correspondence vs registration

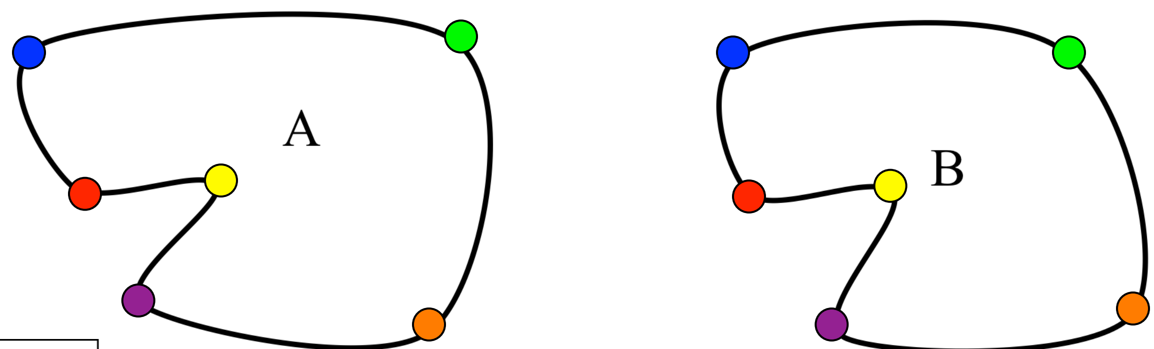
Entropy-based correspondence

- Group-wise
- No need for prior atlas
- No parameterization required
- Efficient
- Generic



Particle Correspondence

- Point-based sampling of the ensemble
  - Same number of particles per shape
- Particles are ordered
  - Ordering implies correspondence



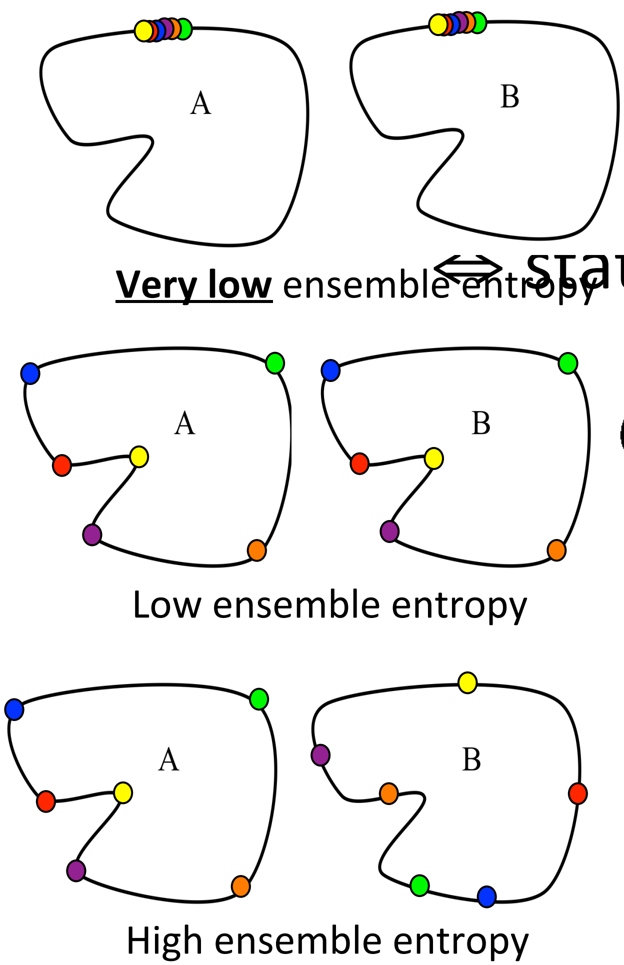
Tradeoff

- Simultaneously maximize both the geometric accuracy and the statistical simplicity of the model

$$Q = \underbrace{H(Z)}_{\text{Ensemble entropy (small = simple)}} - \sum_k \underbrace{H(P^k)}_{\text{Surface entropy (large = accurate)}}$$

k: shape id
P: particle locations
Z: ensemble distribution

Ensemble Entropy



ble entropy

similarity of corresponding points

⇔ statistically compact model

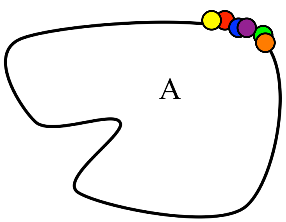
$$Q = H(Z) - \sum_k H(P^k)$$

Ensemble entropy

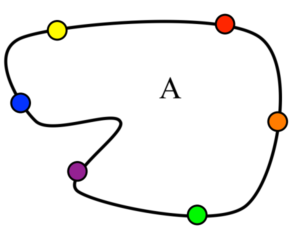
- z represents all sample locations
- Assume normal distribution for z with covariance Σ
- H(Z) ≈ 1/2 log | Σ |
- Σ estimated from data

Surface Entropy

- High surface entropy
  - ⇔ uniform sampling of the surface
  - ⇔ high geometric accuracy



Low surface entropy



High surface entropy

$$Q = H(Z) - \sum_k \underbrace{H(P^k)}_{\text{Surface entropy}}$$

- $H(P^k) = -\int p(x) \log p(x)\, dx$
- Parzen windowing to estimate $p(x_i)$
- $H(P^k) \approx \sum_i \log 1/N(N-1) \sum_{j \neq i} G(x_i - x_j, \sigma_j)$
- Truncated Gaussians

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Generalized Correspondence

- **Main Idea:** anatomical correspondence is rarely a direct result of *spatial proximity* of sample points on the surface

- Allowing correspondence to incorporate *similarity of non-spatial local features*

$$\tilde{P} = f(x_j^k)$$

- Examples of "attributes" f(x)
  - Local curvature
  - Sulcal depth
  - DTI - probabilistic connectivity
  - MRA - distance to vessel

Incorporating attributes

$$Q = \boxed{H(Z)} - \boxed{\sum_k H(P^k)}$$

remains
the same

- Corresponding particles across surfaces should have similar attribute values f(x)

- Particles should be evenly distributed on each surface

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Adaptive sampling

- Surface entropy favors a uniform sampling
- Might miss high-curvature areas
- Solution: adaptive – favor high curvature regions

Uniform $$p(x_i) \approx \frac{1}{N(N-1)} \sum_{j=1, j \neq i}^{N} G(x_i - x_j, \sigma_i)$$

$$Q = H(Z) - \sum_k H(P^k)$$

Surface entropy

Adaptive $$p(x_i) \approx \frac{1}{N(N-1)} \sum_{j=1, j \neq i}^{N} G(\frac{1}{k_j}(x_i - x_j), \sigma_i)$$

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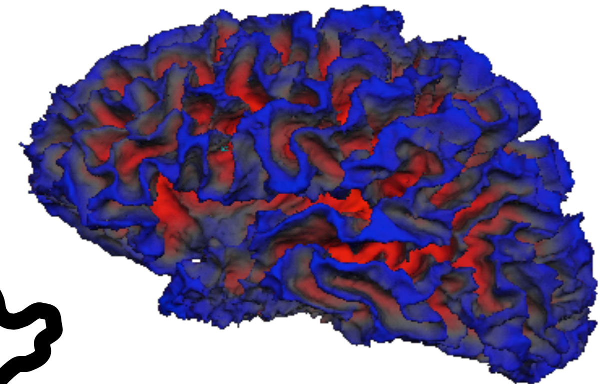
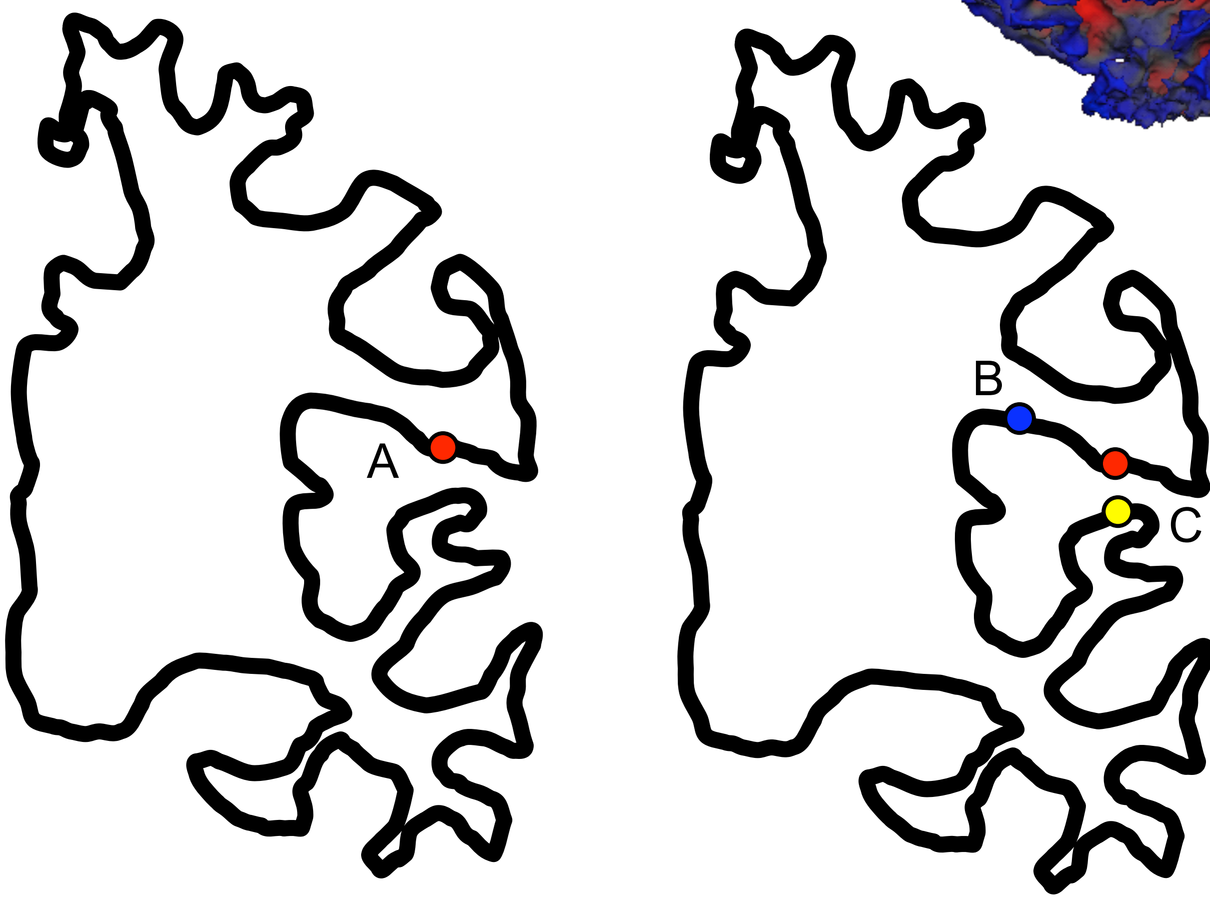
Driving problem: Cortical Correspondence

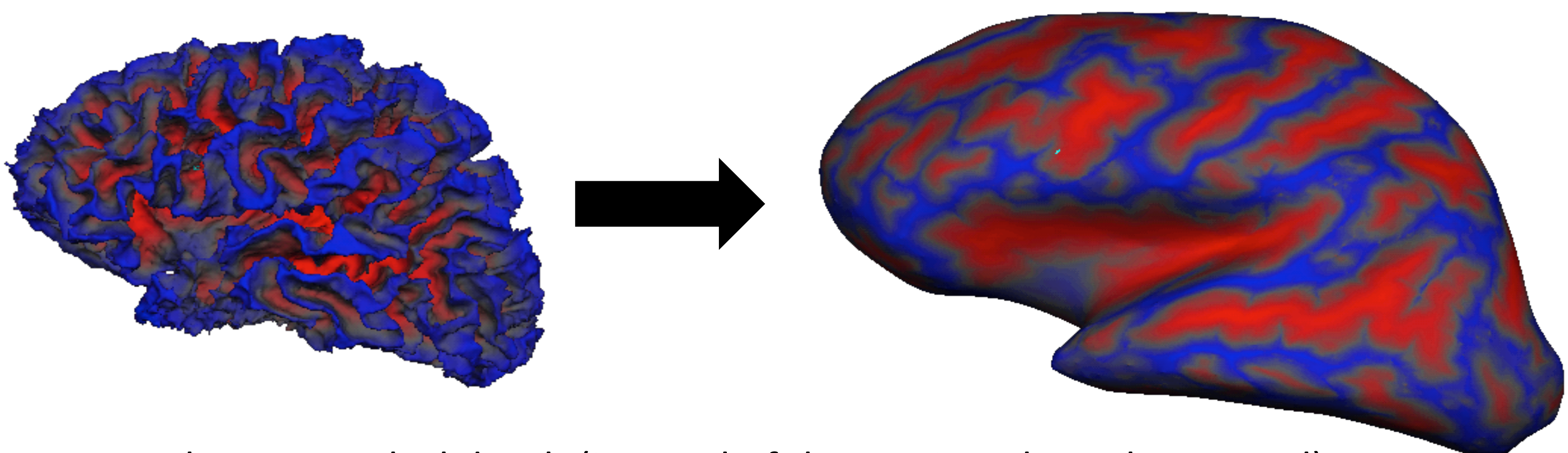
- Highly convoluted surface is a problem





Cortical Correspondence: Dealing with Cortical Geometry

- Highly convoluted surface is a problem
- Our initial solution: Inflate the brain
  - Convex regions move inwards, concave points move outwards
  - Minimizes metric distortion



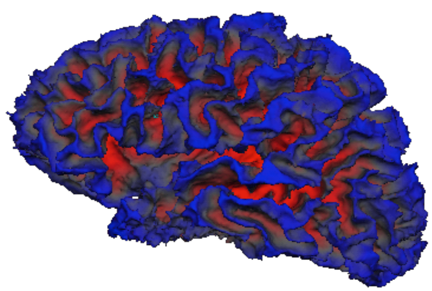
Color map: sulcal depth (integral of the motion along the normal)

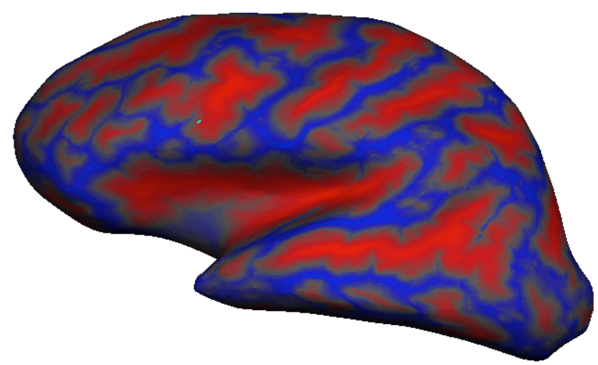
Particle System on the Inflated Cortical Surface

$$Q = H(Z) - \sum_{k} H(P^k)$$

Ensemble entropy
(small = simple)

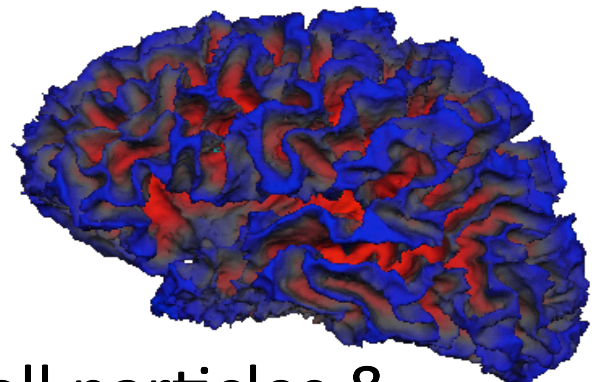
Surface entropy
(large = accurate)





The real solution

- The real problem is using Euclidean distances
- So: use geodesic distances
- Computationally expensive
  - Not computable in closed form
  - Need inter-particle distances for all particles & shapes at every iteration
  - Solution:
    - Precompute distances on a very fine mesh (GPU-based)
    - Interpolate to particle position during optimization



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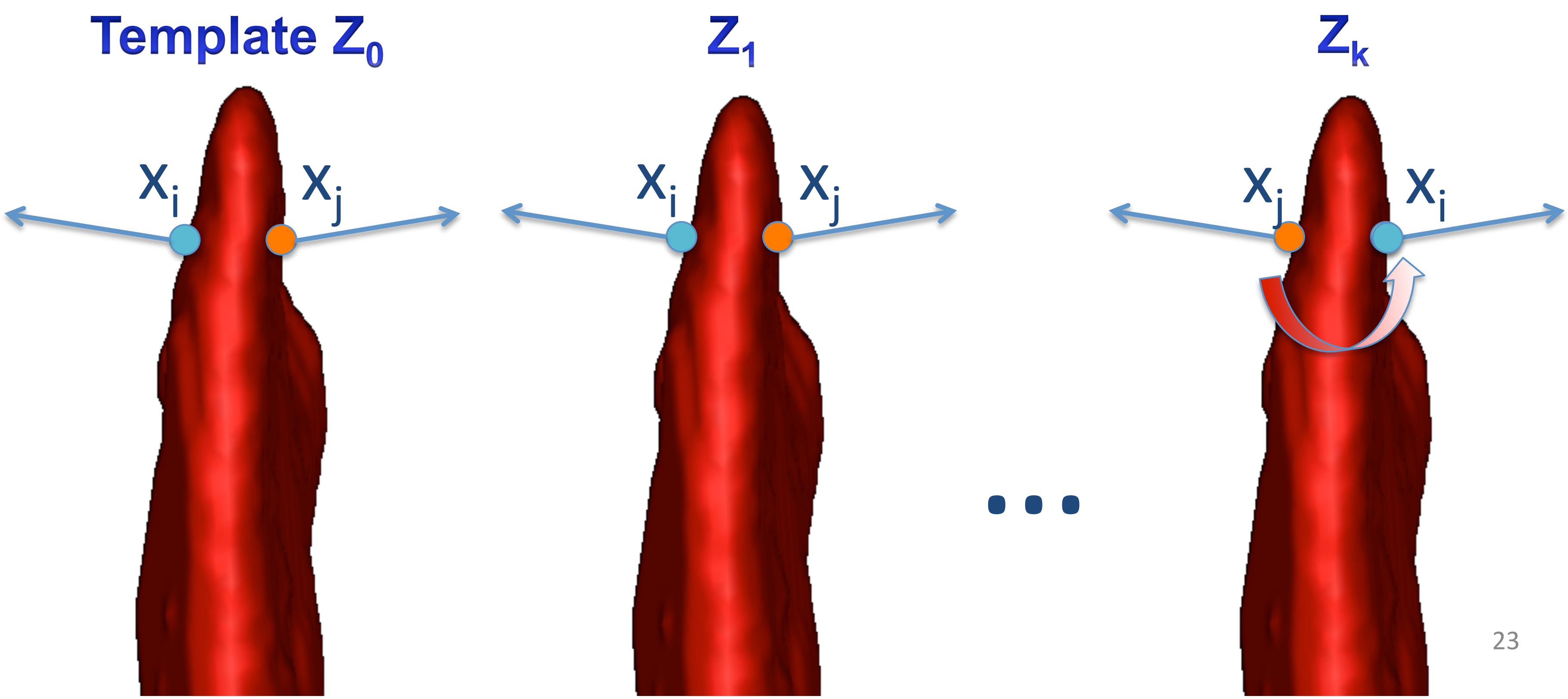
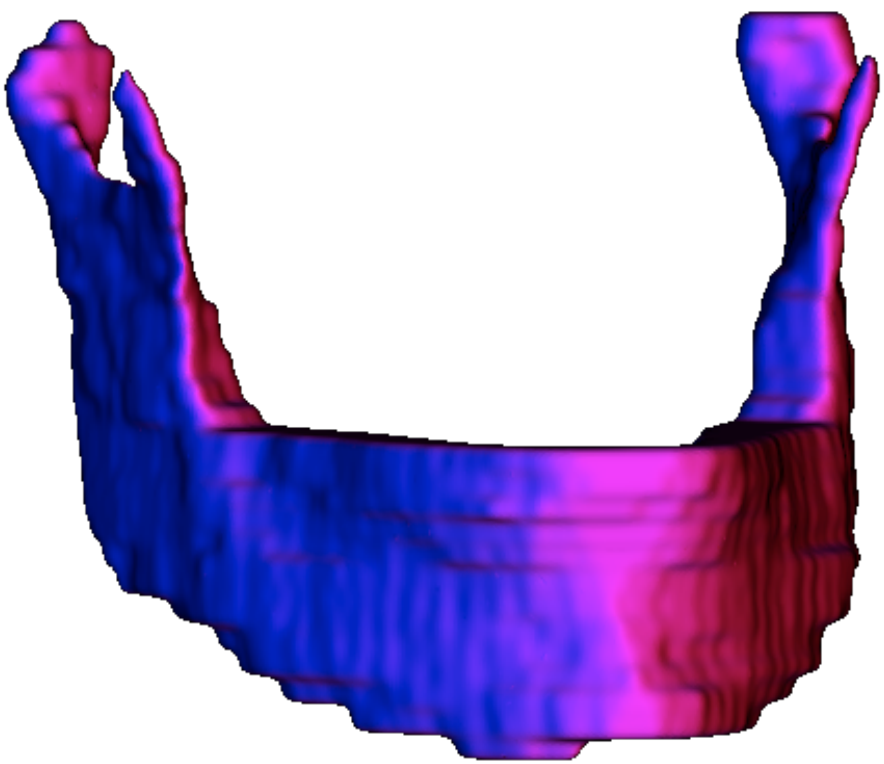
Normal consistency - I

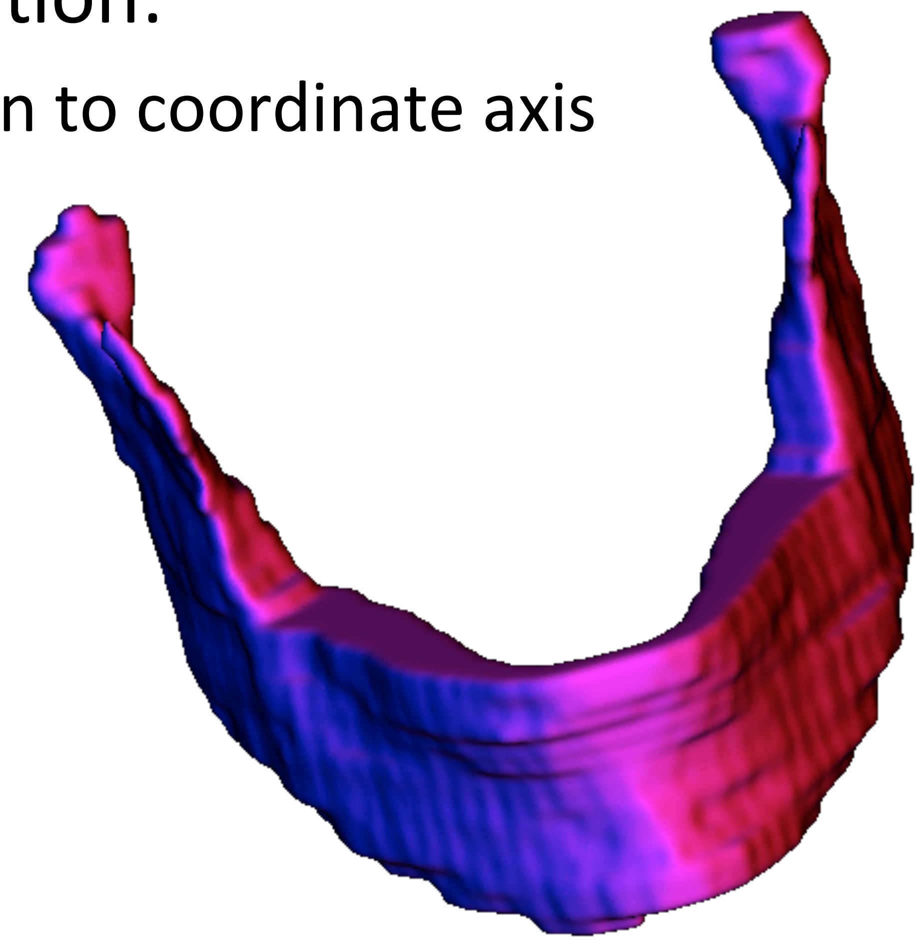
- Quick and dirty solution:
  - Remove particles that switched between opposite sides



Normal consistency - II

- The slightly better solution:
  - Project normal direction to coordinate axis
  - Use as scalar attribute





Normal consistency - III

- Use the full vector

$$Q = H(Z) - \sum_{k=1}^{M} H(P^k) + H(V)$$

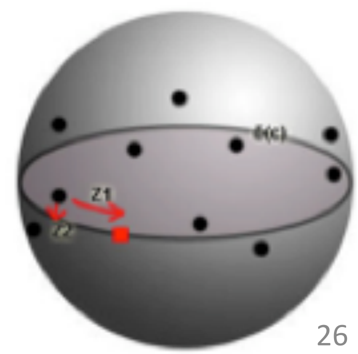
$$H(V) \approx \frac{1}{2} log|\Sigma_V| \approx \frac{1}{2} log|\sum_i \sum_k \hat{n}_i^k \cdot (\hat{n}_i^k)^T|$$

Riemannian distances $\hat{n}_i^k = d(n_i^k, \overline{n}_i)$

Frechet mean

Normal consistency - IV (the saga continues)

- Principal Nested Spheres (PNS)
- Given normals on the unit sphere, fit a great circle
- Find the Frechet mean on the great circle
- Compute principal scores (Z)
- Minimize Z as part of entropy computation



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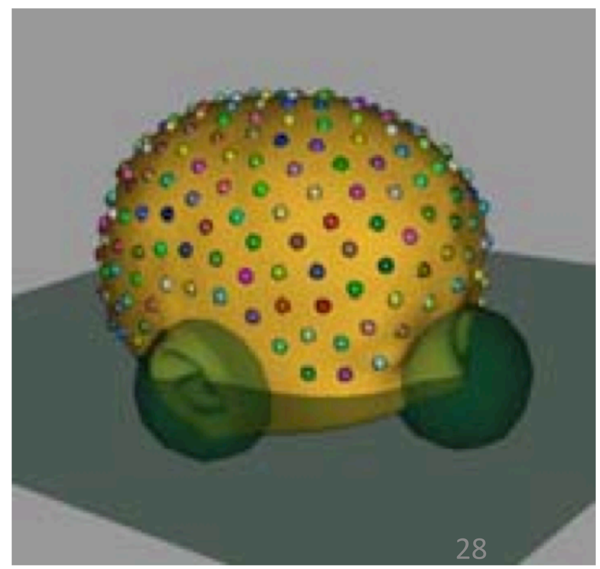
Correspondence on open surfaces

- Basic idea: intersect surface with geometric primitives
- “Virtual” particles on the primitives for surface entropy computation to repel from the boundary



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Multi-object complexes

- Multiple organs per subject
- Simple solution: treat all as one object
- Better solution:
  - Assign object-ID to each particle
  - Constrain each particle to its object
  - Decouple spatial interactions for sampling purposes
  - Shape-space statistics remain coupled

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Linear Regression

- Driving problem: neurodevelopment – shape variability can be explained by age to some extent
- Standard version assumes the model:

$$z = \mu + \epsilon, \epsilon \sim N(0, \Sigma)$$

- For regression, instead use:

$$z = f(t) + \hat{\epsilon}, \hat{\epsilon} \sim N(0, \hat{\Sigma})$$

and minimize entropy associated with $\hat{\epsilon}$ instead of ϵ

- Linear regression: f(t) = a + bt

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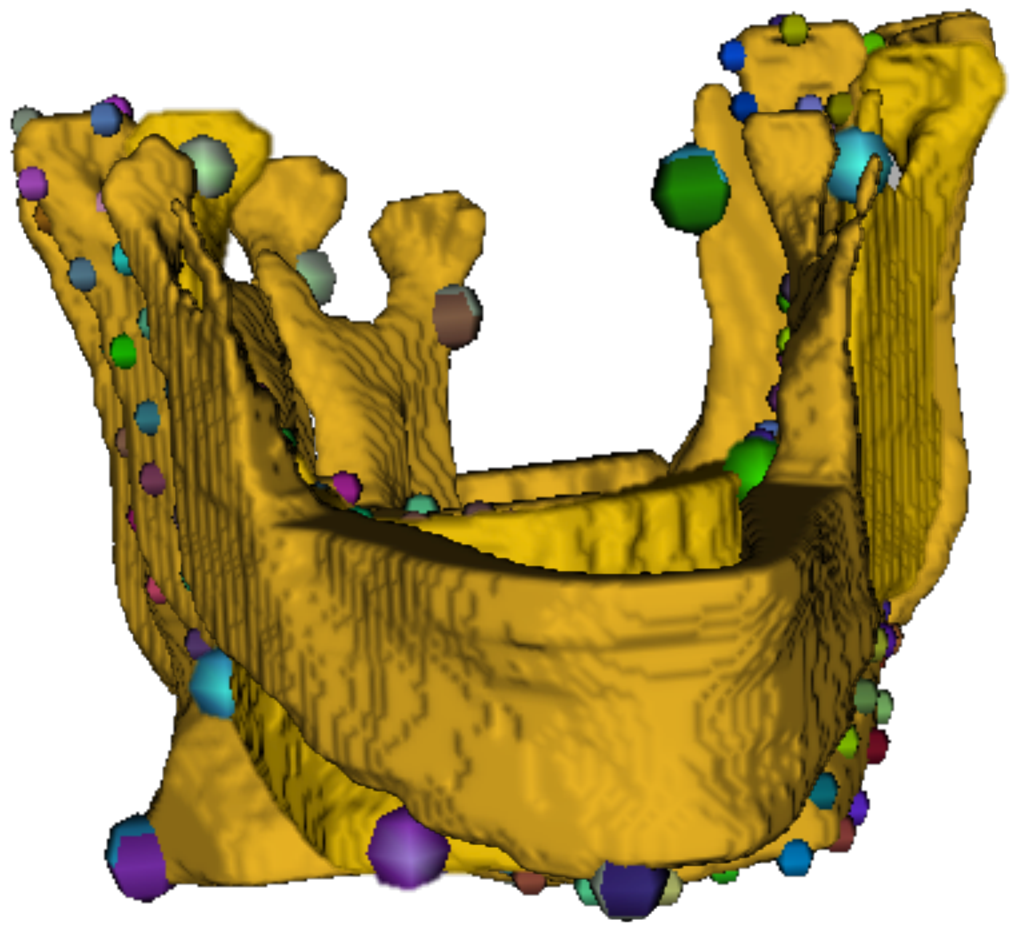
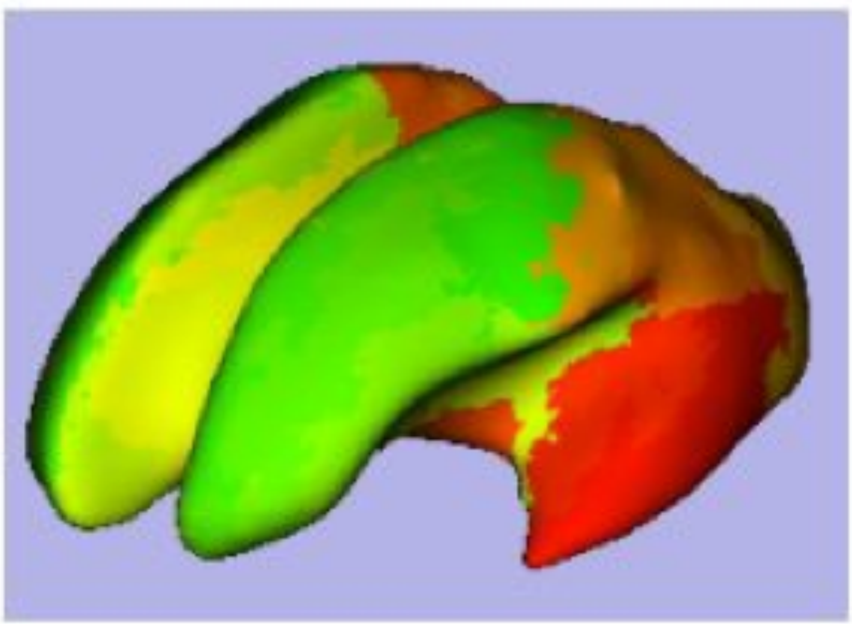
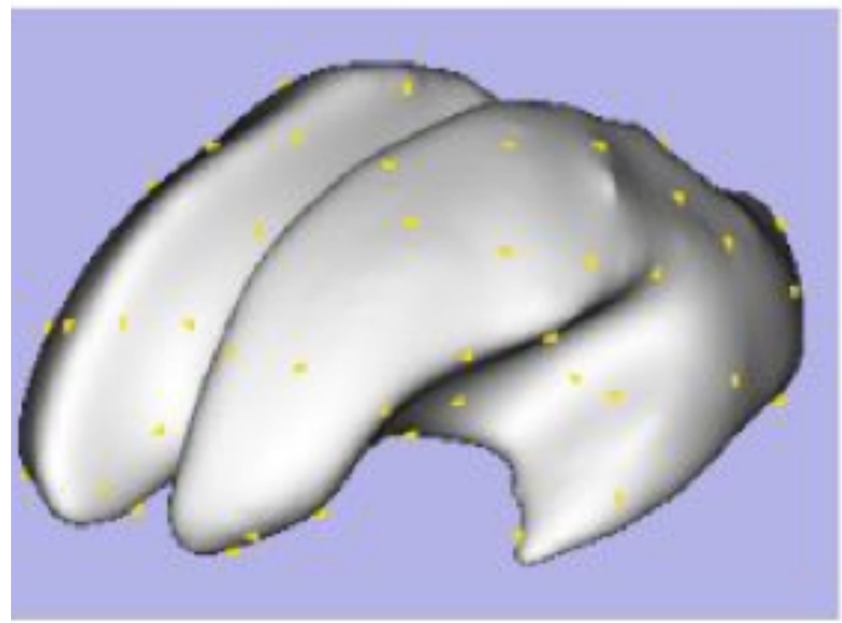
Splitting – initialize particles



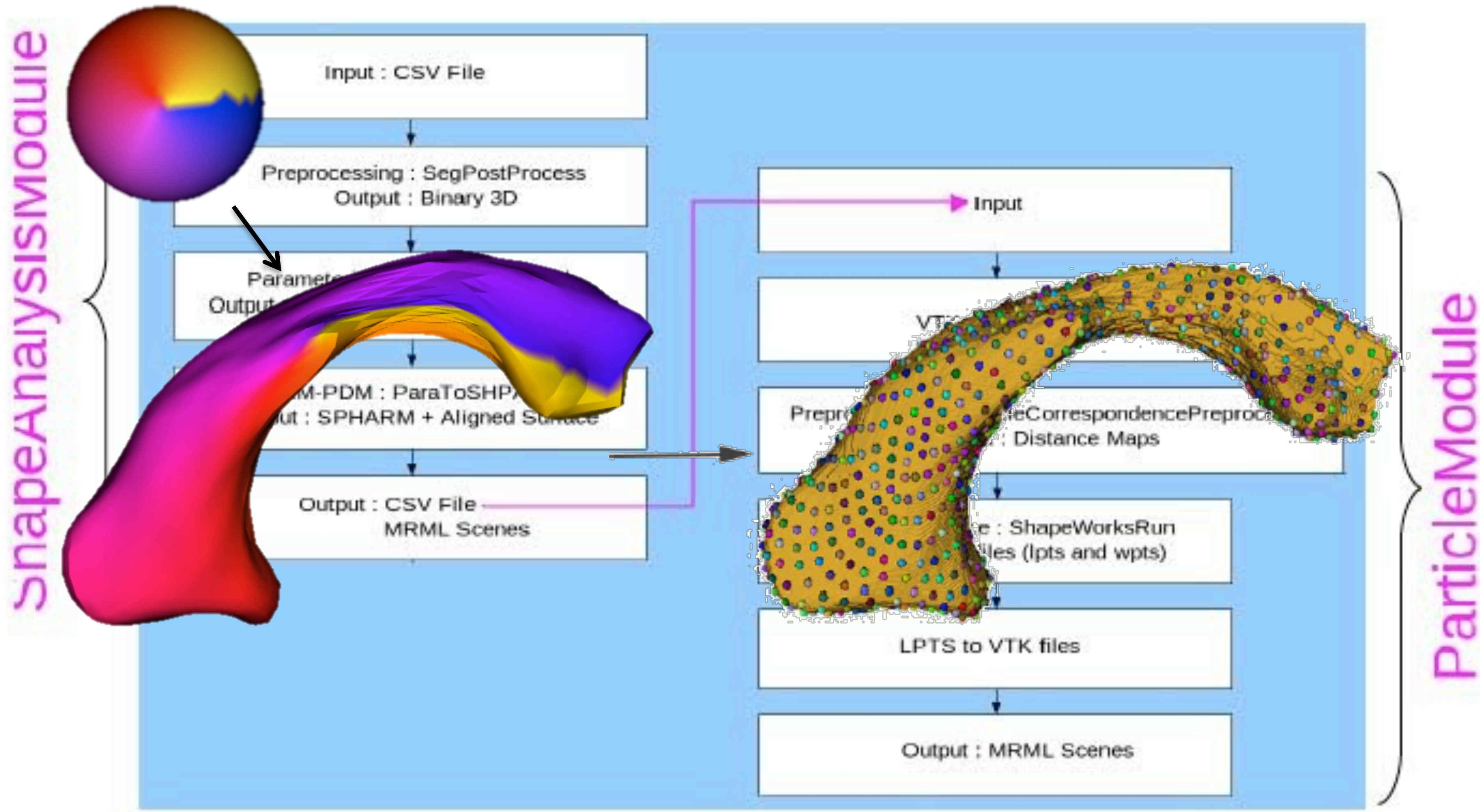
Initialization via parcelation

- Parcellation to speed up, especially when "hard to reach" areas exist



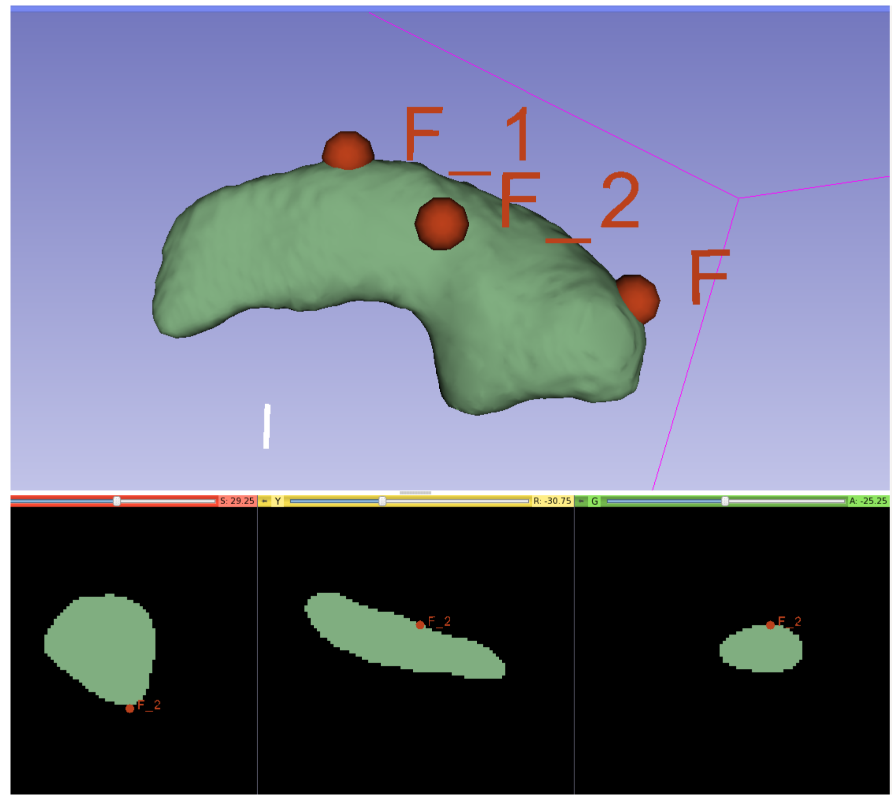


SPHARM+Particles



Manual landmark selection

- Anatomical landmarks
- Easily interfaced with particle sw:
  - Particle files (lpts)
  - Slicer fiducial lists (fcsv)



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ShapeWorks

- http://www.nitrc.org/projects/shapeworks

- Tools
  - ShapeWorksGroom
  - ShapeWorksRun
  - ShapeWorksView
  - ShapeWorksShop
    - ParticleCorrespondencePreprocessing
    - ParticleCorrespondencePostprocessing

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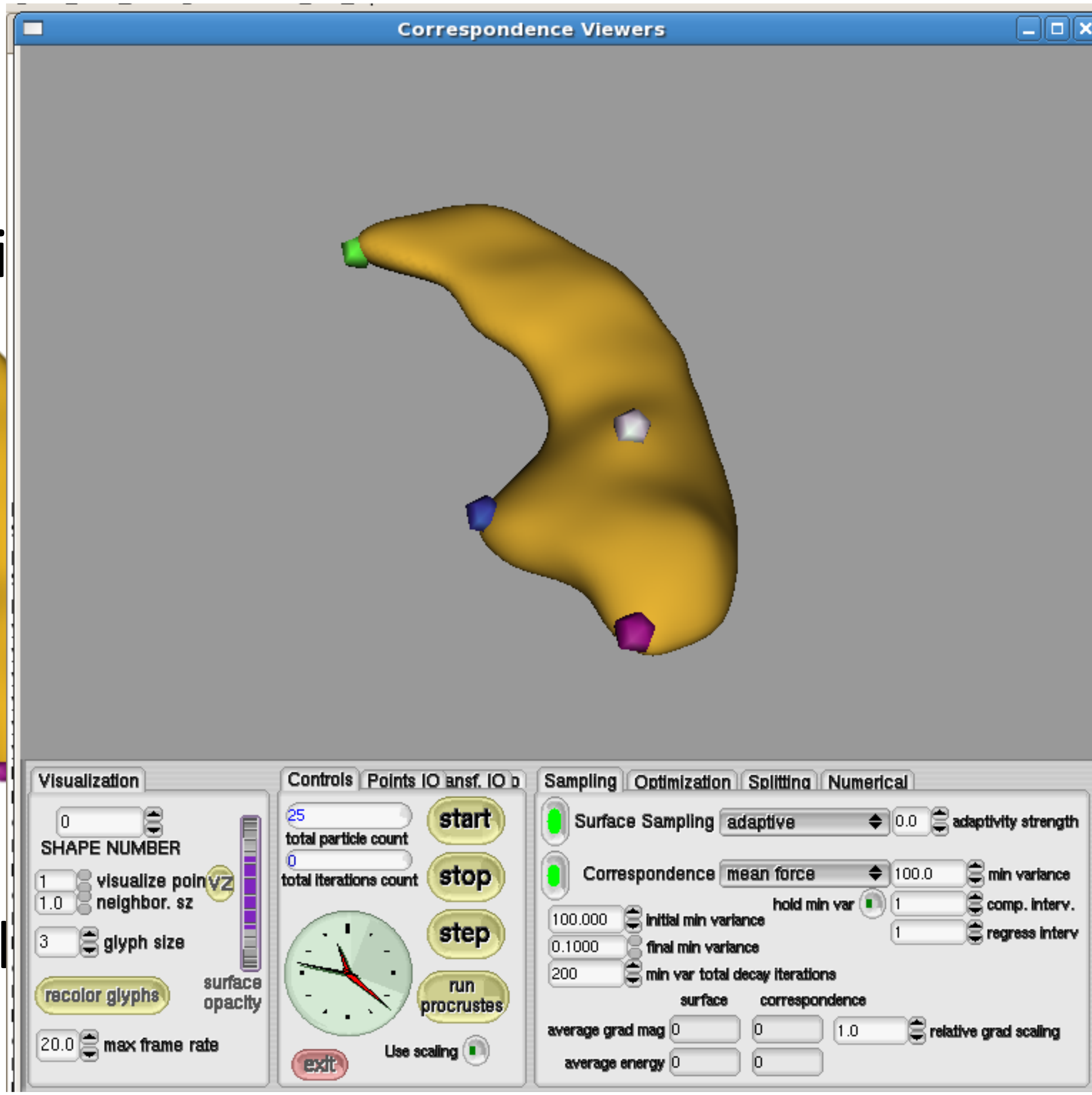
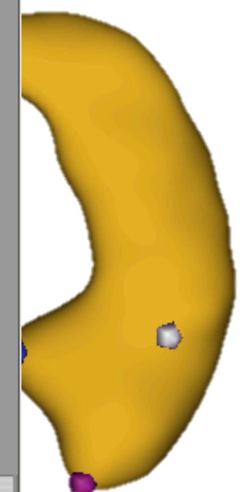


Correspondence Viewers
Visualization
0
SHAPE NUMBER
1
visualize poin
vz
1.0
neighbor. sz
3
glyph size
recolor glyphs
surface opacity
20.0
max frame rate
Controls
Points IO
ansf. IO
25
total particle count
0
total iterations count
start
stop
step
run procrustes
exit
Use scaling
Sampling
Optimization
Splitting
Numerical
Surface Sampling
adaptive
0.0
adaptivity strength
Correspondence
mean force
100.0
min variance
hold min var
1
comp. interv.
1
regress interv
100.000
initial min variance
0.1000
final min variance
200
min var total decay iterations
surface
correspondence
average grad mag
0
0
1.0
relative grad scaling
average energy
0
0



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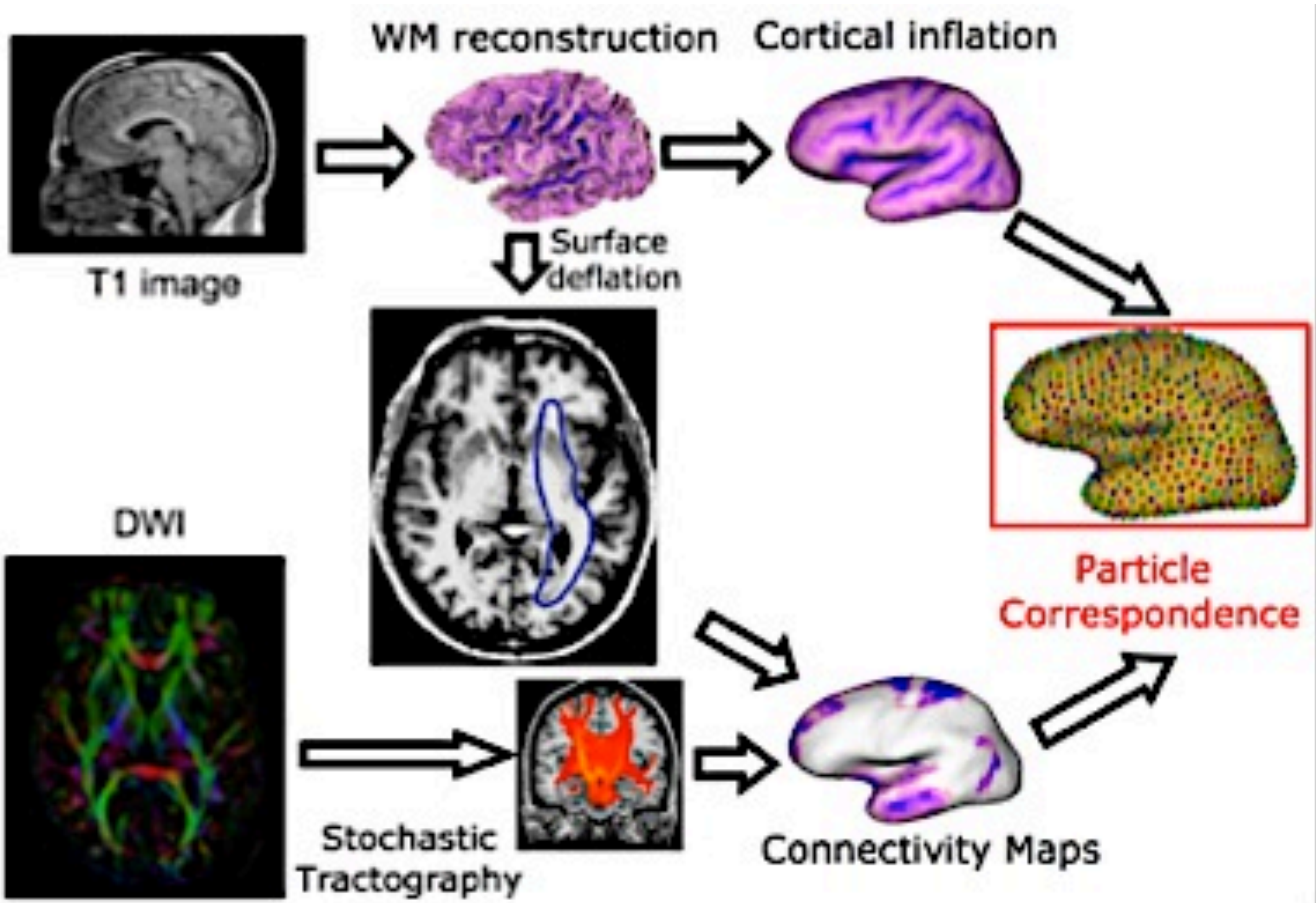
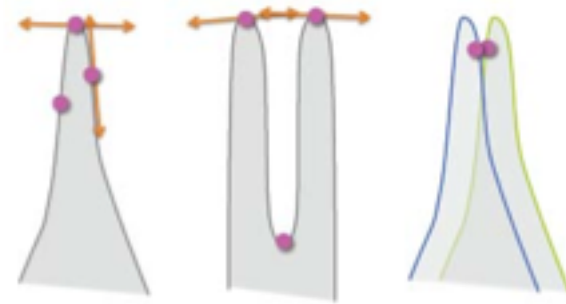
Generalized correspondence

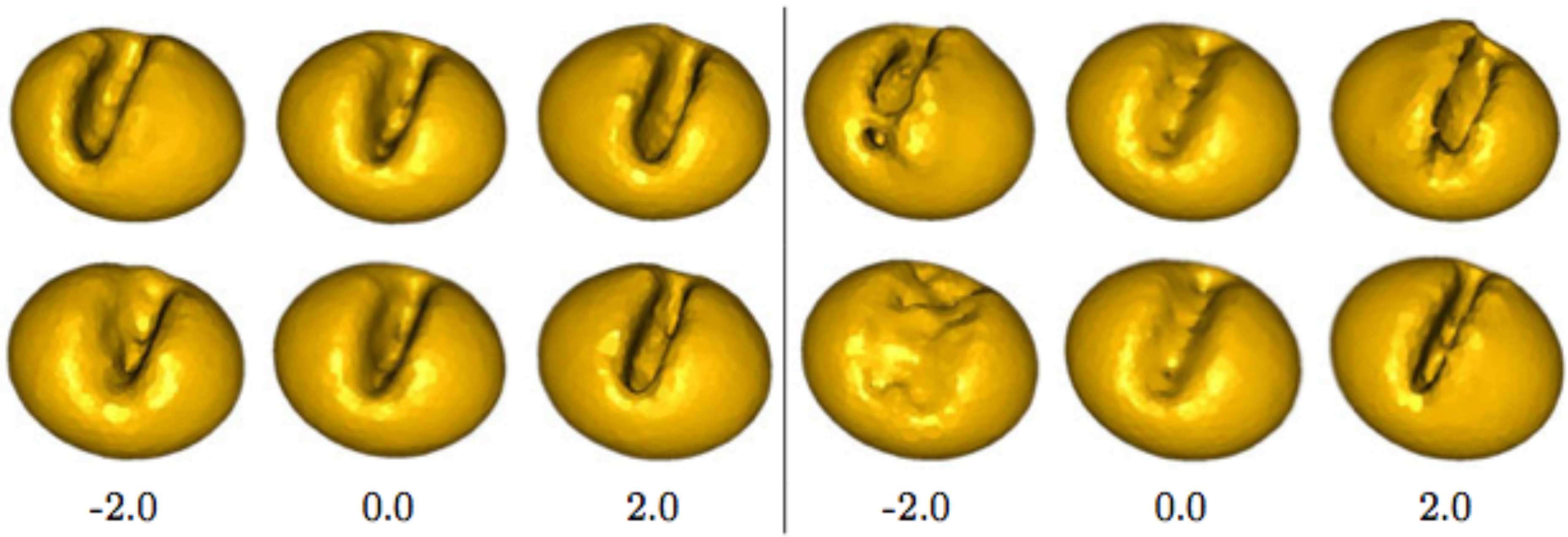


Synthetic data

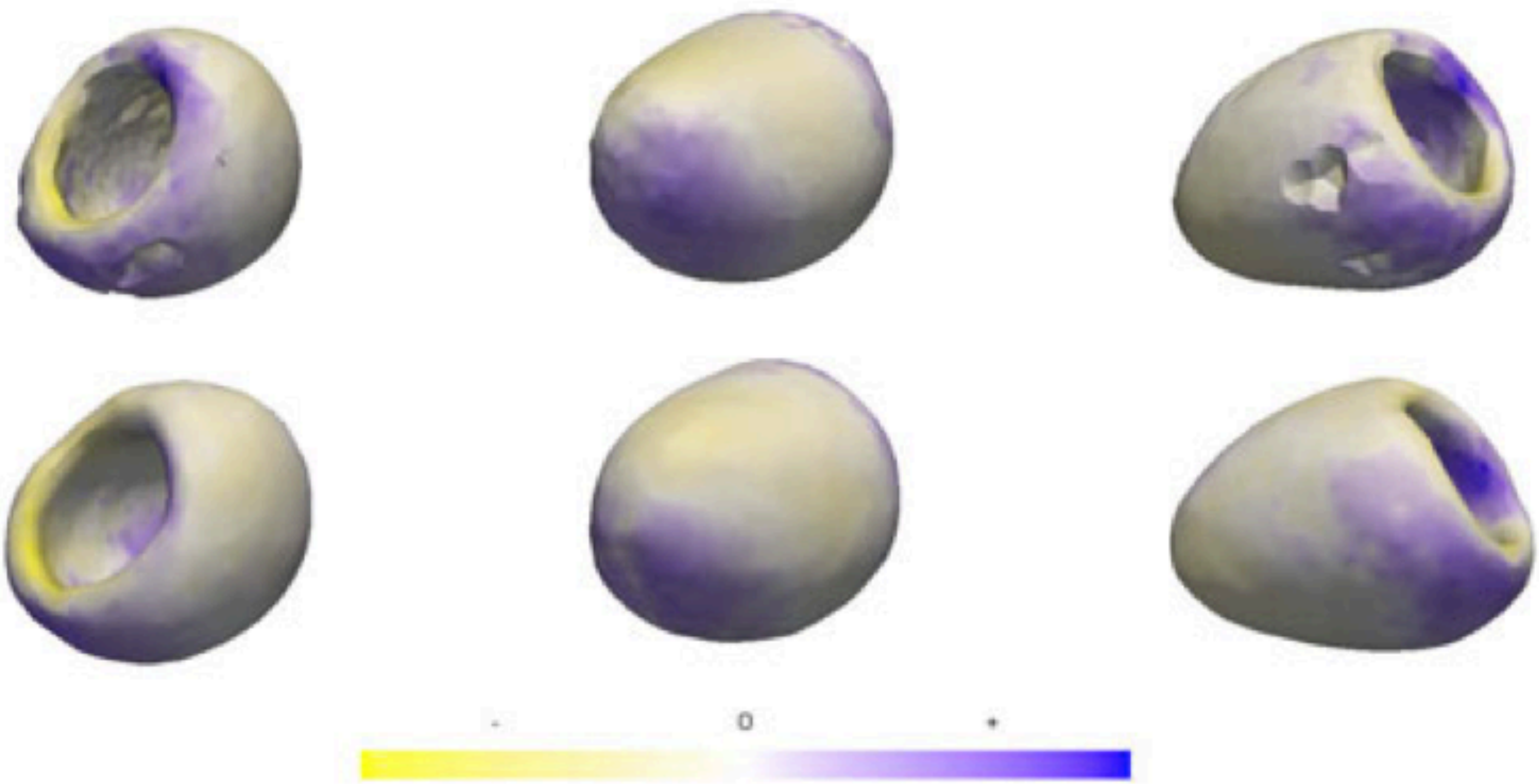
- Geodesic distances

& normal entropy





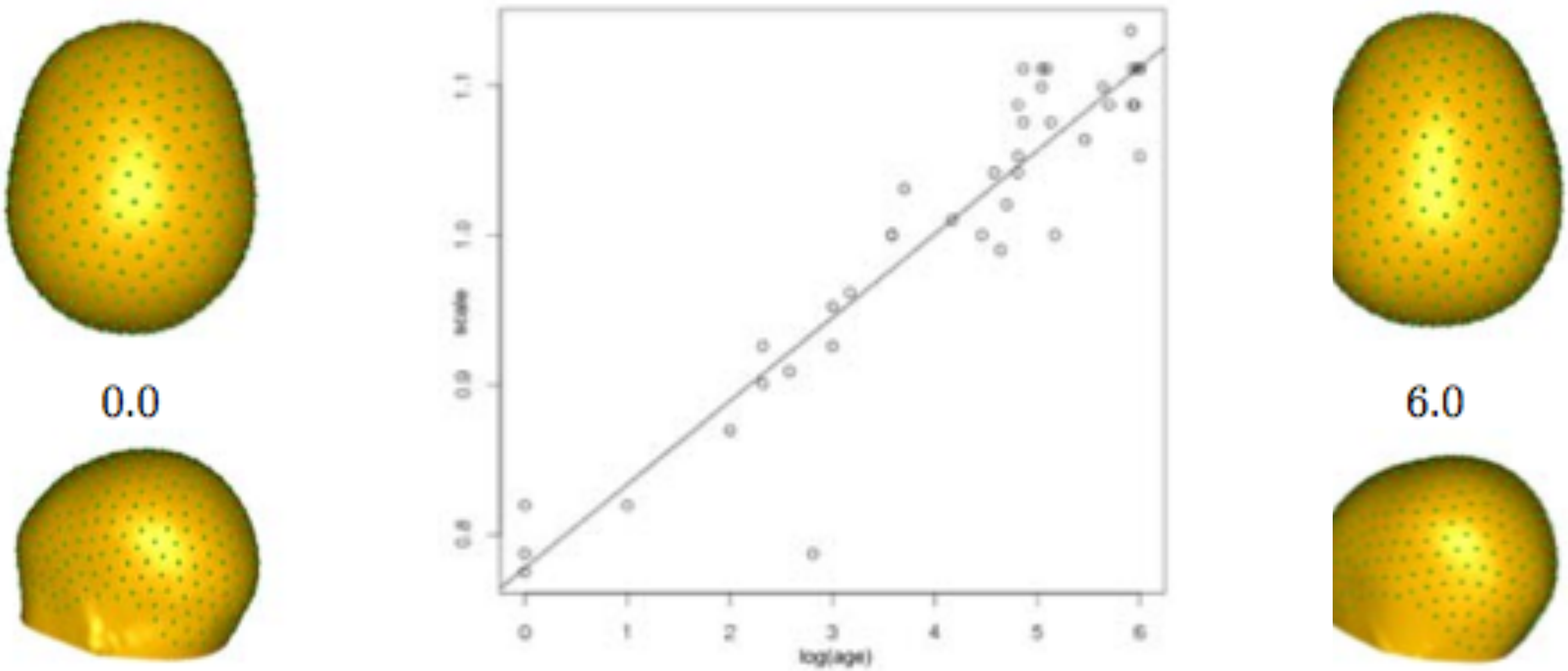
LV myocardium



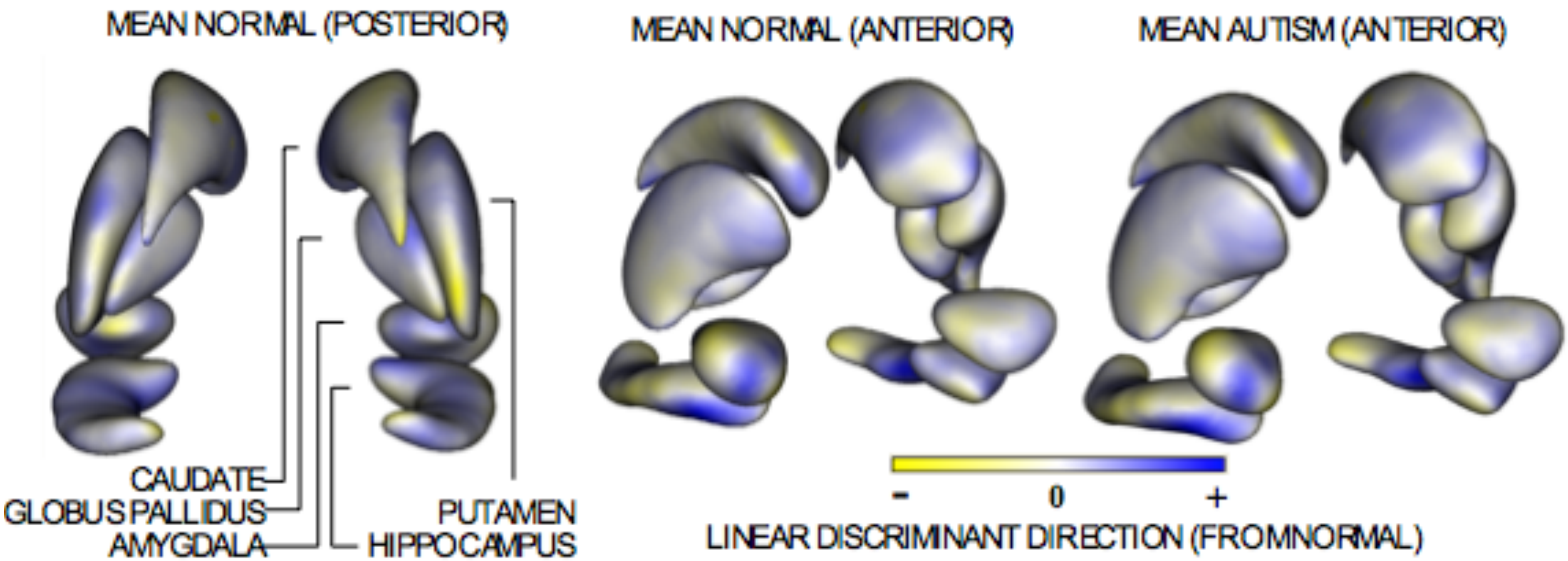
Contraction Expansion

Head shape

- Regression

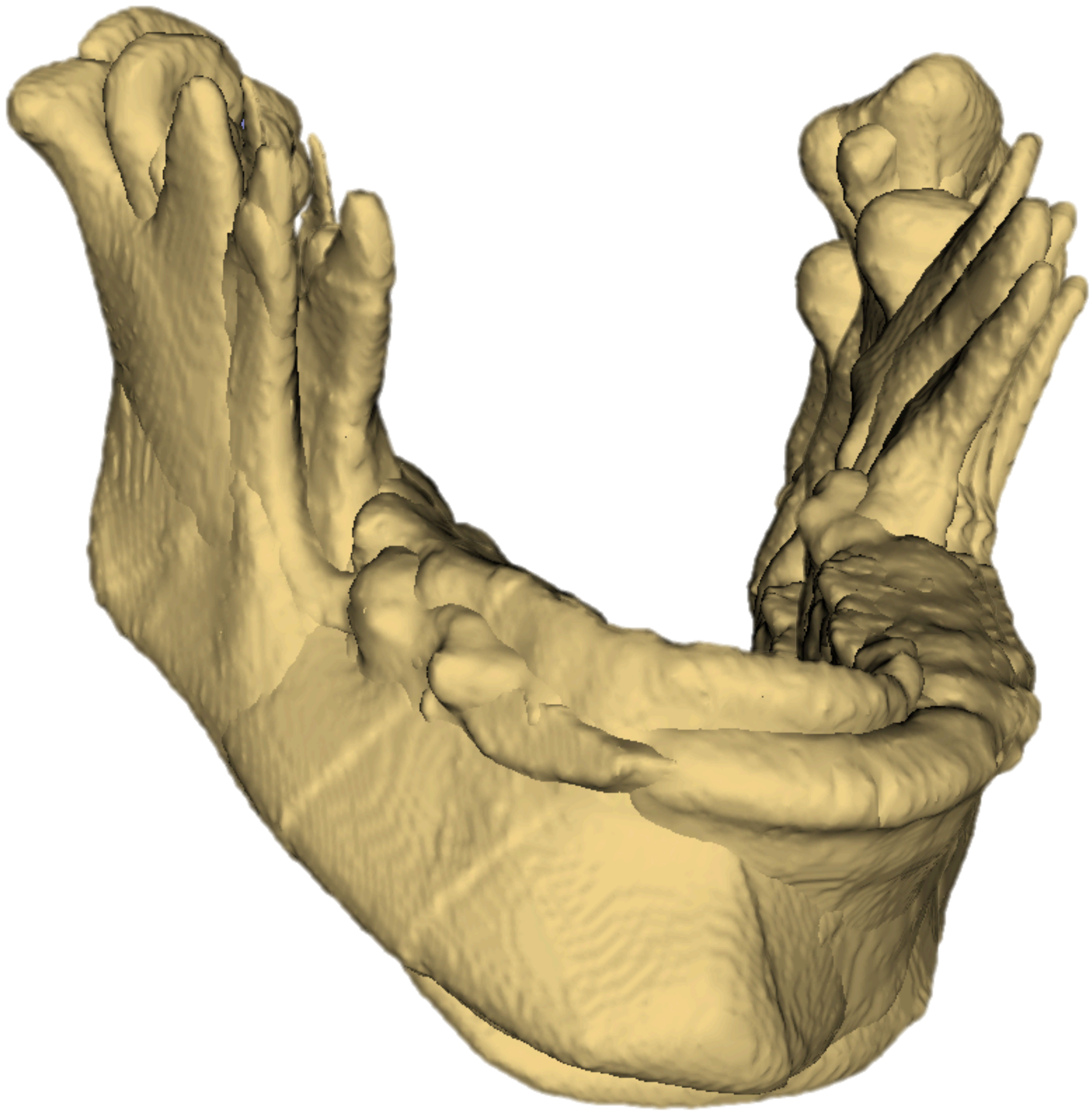


Subcortical shape complexes

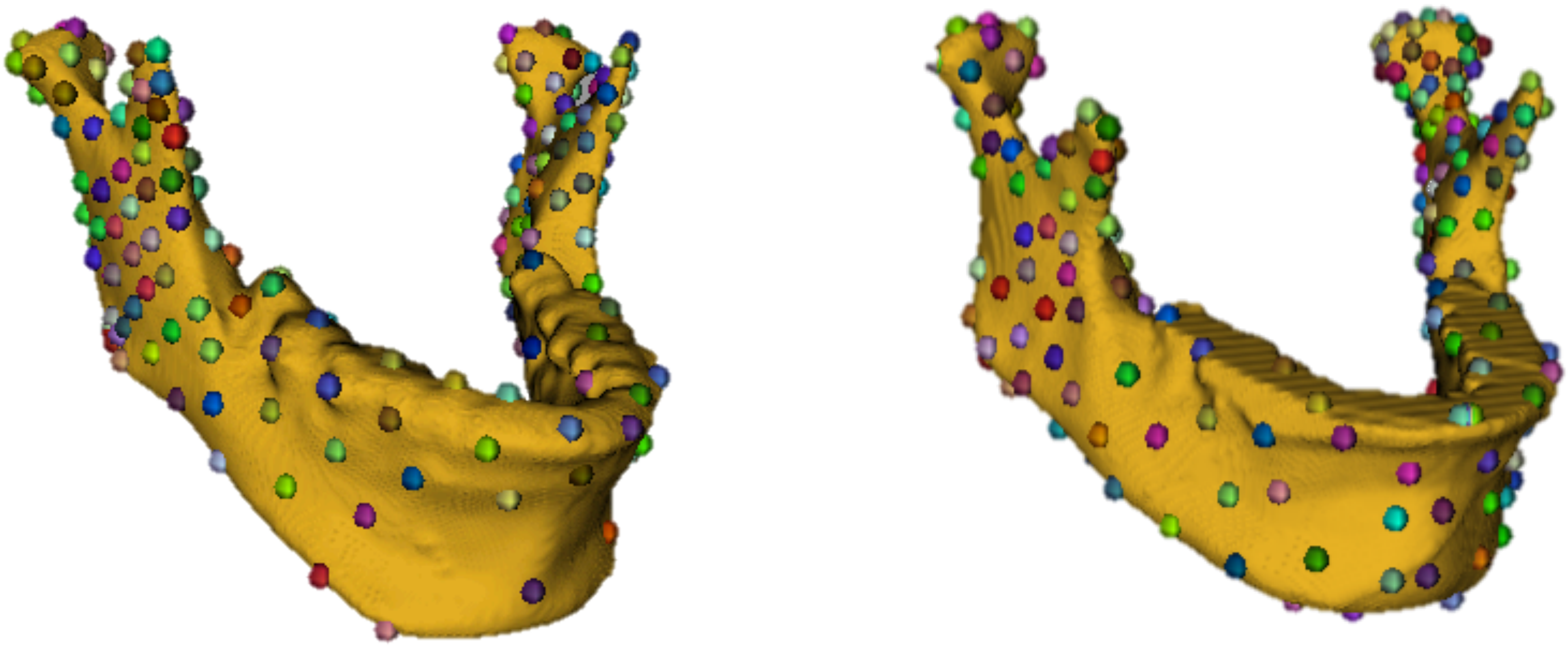


$$\mathbf{w} = (\mathbf{\Sigma_a} + \mathbf{\Sigma_b})^{-1}(\mu_\mathbf{a} - \mu_\mathbf{b})$$

Cranio-facial bones



Cranio-facial bones



Open Discussion

- Problems and pitfalls
  - Initialization vs correspondence

- Future work
  - Journal paper submitted to Computer Vision and Image Understanding – Special Issue in Shape
  - PNS
  - Target interesting applications

References

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